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CONTENTS

UNICURSAL CURVES BY METHOD OF INVERSION,	<i>H. B. Newson</i>
FOREIGN SETTLEMENTS IN KANSAS, - - -	<i>W. H. Carruth</i>
THE GREAT SPIRIT SPRING MOUND, - - -	<i>E. H. S. Bailey</i>
ON PASCAL'S LIMAÇON AND THE CARDIOID, - - -	<i>H. C. Riggs</i>
DIALECT WORD-LIST, - - - - -	<i>W. H. Carruth</i>

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KANSAS UNIVERSITY QUARTERLY.

Vol. I.

OCTOBER, 1892

No. 2.

Unicursal Curves by Method of Inversion.

BY
HENRY BYRON NEWSON.

This paper contains a summary of the work done during the last school year by my class in Modern Geometry. Since many of the results were suggested or entirely wrought out by class-room discussion, it becomes practically impossible to assign to each member of the class his separate portion. Many of the results were contributed by Messrs. M. E. Rice, A. L. Candy, H. C. Riggs, and Miss Annie L. MacKinnon.

The reader who is not familiar with the method of Geometric Inversion should read Townsend's Modern Geometry, chapters IX and XXIV; or a recent monograph entitled, "Das Princep der Reziproken Radien," by C. Wolff, of Erlangen.

When a conic is inverted from a point on the curve, the inverse curve is a nodal, circular cubic.

This is shown analytically as follows: let the equation of the conic be written

$$ax^2 + 2hxy + by^2 + 2gx + 2fy = 0 ;$$

which shows that the origin is a point on the curve. Substituting for

$$x \text{ and } y \quad \frac{x}{x^2 + y^2} \quad \text{and} \quad \frac{y}{x^2 + y^2} ,$$

we have as the equation of the inverse curve

$$ax^2 + 2hxy + by^2 + 2(gx + fy)(x^2 + y^2) = 0.$$

The terms of the second degree show that the origin is a double point on the cubic; and is a crunode, acnode, or cusp, according as the conic is a hyperbola, ellipse, or parabola. The terms of the third degree break up into three linear factors, viz: $gx + fy$, $x + iy$, and $x - iy$, which are the equations of the three lines joining the origin to the three points where the line at infinity cuts the cubic; thus showing that the cubic passes through the imaginary circular points at infinity.

Since the above transformation is rational, it follows that there is a (1, 1) correspondence between the conic and the cubic. This fact is also evident from the nature of the method of inversion. The cubic has its maximum number of double points, viz: one; and hence is unicursal. This unicursal circular cubic may be projected into the most general form of unicursal cubic; the cuspidal variety, however, always remaining cuspidal.

By applying the method of inversion to many of the well known theorems of conics, new theorems are obtained for unicursal, circular cubics. If one of these new theorems states a

projective property, it may at once by the method of projection be extended to all unicursal cubics. Examples will be given below.

The following method of generating a unicursal cubic is often useful. Given two projective pencils of rays having their vertices at A and B; the locus of the intersection of corresponding rays is a conic through A and B. Invert the whole system from A. The pencil through A remains as a whole unchanged, while the pencil through B inverts into a system of co-axial circles through A and B, and the generated conic becomes a circular cubic through A and B, having a node at A. Now project the whole figure and we have the following:—given a system of conics through four fixed points and a pencil of rays projective with it and having its vertex at one of the fixed points, the locus of the intersection of corresponding elements of the two systems is a unicursal cubic, having its node at the vertex of the pencil, and passing through the three other fixed points.

Unicursal cubics are divisible into two distinct varieties, nodal and cuspidal. The nodal variety is a curve of the fourth class and has three points of inflection, one of which is always real. The cuspidal variety is of the third class and has one point of inflection (Salmon, H. P. C., Art. 147). Each of these varieties forms a group projective within itself; that is to say, any nodal cubic may be projected into every other possible nodal cubic, and the same is true with regard to the cuspidal. But a nodal cubic can not be projected into a cuspidal and vice versa.

In applying this method of investigation to the various forms of unicursal cubics and quartics, only a limited number of theorems are given in each case. It will be at once evident that many more theorems might be added, but enough are given in each case to illustrate the method and show the range of its application. It is not necessary to work out all the details, as this paper is intended to be suggestive rather than exhaustive.

NODAL CUBICS.

If an ellipse be inverted from one of its vertices, the inverse curve is symmetrical with respect to the axis; it has one point of inflection at infinity and the asymptote is an inflectional tangent. This asymptote is the inverse of the circle of curvature at the vertex. The cubic has two other points of inflection situated symmetrically with respect to the axis. Hence the three points of inflection lie on a right line, a projective theorem which is consequently true of all nodal cubics. The axis is evidently the harmonic polar of the point of inflection at infinity. Since the axis bisects the angle between the tangents at the node, it follows that the line joining a point of inflection to the node, the two tangents at the node, and the harmonic polar of the point of inflection, form a harmonic pencil. There are three such lines, one to each node, and three harmonic polars; these form a pencil in involution, the tangents at the node being the foci.

Since the asymptote is perpendicular to the axis, we have by projection the following theorem:—through a point of inflection I, draw any line cutting the cubic in B and C. Through P the point of intersection of the harmonic polar and inflectional tangent of I, draw two lines to B and C. The four lines meeting in P form a harmonic pencil. The point of contact of the tangent from I to the cubic is on the harmonic polar of I. Any two inflectional tangents meet on the harmonic polar of the third point of inflection.

The locus of the foot of the perpendicular from the focus of a conic on a tangent is the auxiliary circle. Inverting from the vertex, there are two points, A and B, on the axis of the curve, such that if a circle be drawn through one of them and the node, cutting at right angles a tangent circle through the node, their point of intersection will be on the tangent to the curve where it is cut by the axis. Projecting:—through a point of inflection I of a nodal cubic draw a line cutting the cubic in P and

Q; there are two determinate points on the harmonic polar of I, which have the following property:—draw a conic through P, Q, and the node touching the cubic; draw another conic through one of these points, P, Q, and the node cutting the former, so that their tangents at their point of intersection, together with the lines from it to P and Q form a harmonic pencil; the locus of such a point of intersection is the tangent from I to the cubic.

If three conics circumscribe the same quadrilateral, the common tangent to any two is cut harmonically by the third. Inverting from one of the vertices of the quadrilateral: if three nodal, circular cubics have a common double point and pass through three other fixed points, the common tangent circle through the common node to any two of the cubics is cut harmonically by the third; *i. e.*, so that the pencil from the node to the two points of intersection and the points of contact is harmonic. Projecting this:—given three nodal cubics having a common node and passing through five other fixed points; let a conic be passed through the common node and two of the fixed points, touching two of the cubics. The pencil from the common node to the points of contact and the point where the conic cuts the third cubic is harmonic.

The following theorem may be proved in similar manner:—given a system of cubics having a common node and passing through five other fixed points; let a conic be drawn through the common node and two of the fixed points; the lines drawn from the points where it cuts the cubics to the common node form a pencil in involution.

A variable chord drawn through a fixed point P to a conic subtends a pencil in involution at any point O on the conic. Inverting from O:—a system of circles through the double point of a nodal circular cubic and any other fixed point P, is cut by the cubic in pairs of points which determine at the node a pencil in involution. Projecting:—a system of conics through the node of a unicursal cubic, two fixed points on the curve, and any fourth fixed point, is cut by the cubic in pairs of points which determine at the node a pencil in involution.

We give another proof of the theorem that the three points of inflection of a nodal cubic lie on a right line. This is easily shown by inversion and is a beautiful example of the method.

There are three points on a conic whose osculating circles pass through a given point on the conic; these three points lie on a circle passing through the given point.^[1] (Salmon's Conics, Art. 244, Ex. 5.) By inverting from the given point and then projecting, we readily see that there are three points of inflection on a nodal cubic which lie on a right line. If the above conic be an ellipse, the three osculating circles are all real; but if it be a hyperbola, one only is real. Hence an acnodal cubic has three real points of inflection, while a crunodal one has one real and two imaginary.

The reciprocals of many of the theorems of this section are of interest and will be given under Quartics.

CUSPIDAL CUBICS.^[2]

Inverting the parabola from its vertex we obtain the Cissoid of Diocles. The focus of the parabola inverts into a point on the cuspidal tangent which I shall call the focus of the cissoid. The circle of curvature at the vertex of the parabola inverts into the asymptote of the cissoid. This asymptote is also plainly the inflectional tangent, and the point at infinity is the point of inflection. The directrix of the parabola inverts into a circle through the cusp of the cissoid having the cuspidal tangent for a diameter. Hall calls this the directrix circle. The double ordinate of the parabola which is tangent to the circle of curvature of the vertex inverts into the circle usually called the base circle of the cissoid.^[3]

The cissoid may fairly be called the simplest form of the cuspidal cubic. Its projection and polar reciprocal are both cuspidal cubics. I shall now deduce from the parabola a few simple propositions for the cissoid, and then extend them to all cuspidal cubics.

(1) It is known that the locus of the intersection of tangents to the parabola which are at right angles to one another, is the directrix. Inverting:—the locus of the intersection of tangent circles to the cissoid through the cusp and at right angles to each other is the directrix circle.

(2) For the parabola, two right lines OP and OQ , are drawn through the vertex of the parabola at right angles to one another, meeting the curve in P and Q ; the line PQ cuts the axis at a fixed point, whose abscissa is equal to its ordinate. Inverting:—two right lines, OP and OQ , are drawn at right angles to one another through the cusp of the cissoid, meeting the curve in P and Q ; the circle OPQ passes through the intersection of the axis and asymptote.

(3) If the normals at the points P, O, R , of a parabola meet at a point, the circle through POR will pass through the vertex. Inverting:—through a fixed point and the cusp of a cissoid, three and only three circles can be passed, cutting the cissoid at right angles; these three points of intersection are collinear.

From the geometry of the cissoid we see that if any line be drawn parallel to the asymptote, cutting the curve in two points, B and C , the segment BC is bisected by the axis. Hence, projecting the curve we have the following theorem:—any line drawn through the point of inflection is cut harmonically by the point of inflection, the curve, and the cuspidal tangent. Thus the cuspidal tangent is the harmonic polar of the point of inflection. The polar reciprocal of this last theorem reads as follows:—if from any point on the cuspidal tangent the two other tangent lines be drawn to the curve, and a line to the point of inflection, these four lines form a harmonic pencil. These are fundamental propositions in the theory of cuspidal cubics.

(4) Projecting proposition (1) above, we have the generalized theorem:—through the point of inflection draw any line cutting the cubic in B and C ; through B, C , and the cusp draw two conics tangent to the cubic, and intersecting in a fourth point such that the two tangents to the conics at their point of intersection, together with the two lines from it to B and C , form a harmonic pencil; the locus of all such intersections is a conic through B, C , and the cusp having the point of inflection and the cuspidal tangent for pole and polar.

(5) Reciprocating (4) we have:—through any point on the cuspidal tangent draw the two other tangents, B and C , to the cubic. Touching B, C , and the inflectional tangent draw two conics, such that the points of contact of their common tangent, together with the points where their common tangent cuts the tangents B and C , form a harmonic range; the envelope of such common tangents is a conic having the cuspidal tangent and the point of inflection for polar and pole.

(6) Projecting (2) we obtain the following:—through the point of inflection draw any line cutting the curve in B and C ; take any other two points on the cubic such that the pencil from the cusp, $O, O(BPCQ)$ is harmonic; the conic passing through O, B, P, C, Q will pass through the intersection of the cuspidal and inflectional tangents.

(7) Reciprocating (6):—from any point on the cuspidal tangent draw two other tangents, B and C , to the cubic; take any two other tangents, P and Q , such that the range cut from the inflectional tangent by B, C, P, Q , is harmonic; the conic touching B, C, P, Q , and the inflectional tangent will also touch the line joining the point of inflection and the cusp.

(8) Projecting (3):—through the point of inflection draw any line cutting the cubic in B and C ; through the cusp O and the points B and C on the cubic and any other fixed point P , three, and only

three, conics can be passed, such that the tangent to the conic and cubic at their remaining point of intersection, together with the lines from it to B and C, form a harmonic pencil; these three points of intersection are collinear.

SYSTEMS OF CUBICS THROUGH NINE POINTS.

Let U and V be the equations of two given cubics, then $U + kV$ is the equation of a system of cubics through their nine points of intersection. Twelve cubics of this system are unicursal, and the twelve nodes are called the twelve critic centres of the system. (See Salmon's H. P. C., Art. 190.)

Let the equation of the system be written briefly

$$a + ka_1 + (b + kb_1)x + (c + kc_1)y + u_2 + u_3 = 0 ;$$

one, and only one, value of k makes the absolute term vanish; hence one, and only one, curve of the system passes through the origin, which may be any point in the plane. Make the equation of the system homogeneous by means of z, and differentiate twice with respect to z; we obtain thus the equations of the polar conics and polar lines of the origin with respect to the system.

The polar conics of the origin are given by

$$3(a + ka_1) + 2\{ (b + kb_1)x + (c + kc_1)y \} + u_2 = 0 ;$$

thus showing that the polar conics of any point, with respect to the system of cubics, form a system through four points. The polar lines of the origin are given by

$$3(a + ka_1) + (b + kb_1)x + (c + kc_1)y = 0 ,$$

which represents a pencil of lines through a point.

Suppose now the origin to be at one of the critic centres; then for a particular value, k_1 , all terms lower than the second degree must vanish, so that

$$\begin{vmatrix} a & b & c \\ a_1 & b_1 & c_1 \end{vmatrix} = 0.$$

The factors of the terms of u_2 , which involves k_1 , represent the tangents at the double point to the nodal cubic, and also the polar conic of the origin with respect to this nodal cubic. Hence a critic centre is at one of the vertices of the self-polar triangle of its system of polar conics. The opposite side of this triangle is the common polar line of the critic centre with respect to its system of polar conics, and hence it is also the common polar line of the critic centre with respect to the system of cubics. The four basal points of the system of polar conics lie two and two upon the tangents at the double point of the nodal cubic.

When the origin is taken at one of the nine basal points of the system of cubics, a and a_1 both vanish. Hence it is readily seen that a basal point of a system of cubics is also a basal point of its system of polar conics and the vertex of its pencil of polar lines.

Suppose two of the basal points of the system of cubics to coincide, then every cubic of the system, in order to pass through two coincident points, must touch a common tangent at a fixed

point. The common tangent is the common polar of its point of contact, both with respect to the system of cubics and to its system of polar conics. Hence the union of two basal points gives rise to a critic centre. The self-polar triangle of its system of polar conics here reduces to a limited portion of the common tangent. This line is not a tangent to the nodal cubic, but only passes through its double point.

Suppose three of the basal points of a system of cubics to coincide, such a point will then be a point of inflection on each cubic of the system. For, in the last case, if a line be drawn from the point of contact of the common tangent to a third basal point of the system, such a line will be a common chord of the system of cubics. Suppose, now, this third basal point be moved along the curves until it coincides with the other two; then the common chord becomes a common tangent, which cuts every cubic of the system in three coincident points, and hence is a common inflectional tangent.

Since the polar conic of a point of inflection on a cubic consists of the inflectional tangent and the harmonic polar of the point, and since the polar conics of a fixed point with respect to a system of cubics pass through four fixed points, it follows that in a system of cubics having a common point of inflection and a common inflectional tangent the harmonic polars of the common point of inflection meet in a point.

Since the common inflectional tangent is the common polar line of the common point of inflection, it follows that such a point is a critic centre of the system of cubics. One cubic of the system then has a node at the common point of inflection of the system, and forms an exception. The line which is the common inflectional tangent to the other cubics of the system cuts this also in three points, but is one of the tangents at the double point; the other tangent at the double point goes through the vertex of the pencil of harmonic polars.

It is evident that the nine basal points of a system of conics may unite into three groups of three each. The cubics will then all have three common points of inflection, and at these points three common inflectional tangents. These three points all lie on a line.

When four basal points of the system of cubics coincide, such a point is a double point on every cubic of the system. This is easily shown as follows, using the method of inversion. Let a system of conics through four points be inverted from one of the four points. The system of conics inverts into a system of cubics, having a common node and passing through three other finite fixed points and the two circular points at infinity. Since the common node counts as four points of intersection, it follows that any two cubics of the system, and hence all of them, intersect in nine points. This system can be projected into a system having a common double point and passing through any five other fixed points.

A number of theorems concerning the system of cubics can easily be inferred from known theorems concerning the system of conics. Since two conics of the system are parabolas, it follows that two cubics of the system are cuspidal. Since three conics of the system break up into pairs of right lines, it follows that three cubics of the system break up into a right line and a conic. Each right line and its corresponding conic intersect in the common double point. The line at infinity cuts the system of conics in pairs of points in involution, the points of contact of the two parabolas of the system being the foci; it follows on inversion that the pairs of tangents to the cubics at their common node form a pencil in involution, the two cuspidal tangents being the foci.

If the four basal points of the system of conics lie on a circle, this circle inverts into a right line, and one cubic then consists of this right line and the lines joining the centre of inversion to the circular points at infinity. This theorem may be stated for the system of cubics as follows: if the

conic determined by the five basal points of the system of cubics (not counting the common double points), break up into right lines, the line passing through three of the five points, together with the lines joining the other two points to the common node, constitute a cubic of the system.

If three of the four basal points of the system of conics lie on a line, the conics consist of this line and a pencil of lines through the fourth basal point. Inverting from this fourth point and then projecting, we have a system of cubics consisting of a pencil of lines and a conic through the vertex and the four other fixed points. Hence, when the five fixed points of such a system of cubics lie on a conic through the common node, this conic is a part of every cubic of the system. If we invert the above system of conics from one of the three points on the right line, and then project, we obtain a system of cubics which consists of a system of conics through four fixed points, and a fixed right line through one of these four points. Hence, if two of the five basal points of such a system of cubics be on a line through the common node, this line is a part of every cubic of the system.

If a system of conics having one basal point at infinity be inverted from one of the remaining basal points, this point at infinity inverts to the center of inversion, and we obtain a system of cubics having five coincident basal points and hence passing through only four others. The system of cubics is now so arranged that one tangent at their common double point is common to all. Only one cubic of the system is cuspidal. As before three cubics break up into a right line and conic.

If two of three basal points of the system of conics be at infinity, the system of cubics obtained by projection and inversion has six coincident basal points and hence only three others. This system has both tangents at the common node common to all cubics of the system. If the two basal points at infinity in the system of conics be coincident, all the conics are parabolas, and hence all the cubics of the system are cuspidal and have a common cuspidal tangent.

If three of the basal points of the system of conics be at infinity, the conics consist of the line at infinity and a pencil of lines through the finite basal point. Inverting from the latter, we obtain a system of cubics with seven coincident basal points. This system is made up of a pencil of lines meeting in the seven coincident basal points together with the two lines joining this to the other two basal points of the system. These two lines are part of every cubic of the system.

If one of the remaining basal points be moved up to join the seven coincident ones, one of these fixed lines becomes indeterminate, and the system of cubics through eight coincident points consists of a fixed line through the eight coincident points and the ninth fixed point together with any two lines of the pencil through the eight points. If the nine basal points coincide, any three lines through it form a cubic of the system.

UNICURSAL QUARTICS.

The inverse of a conic from any point not on the curve is a nodal bicircular quartic. This is shown by inverting the general equation of the conic

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 ;$$

by substituting for x and y _____ and _____ ,

$$x^2 + y^2 \quad x^2 + y^2$$

we get the equation

$$ax^2 + 2hxy + by^2 + 2(gx + fy)(x^2 + y^2) + c(x^2 + y^2)^2 = 0.$$

The origin is evidently a double point on the curve, and is a crunode, acnode, or cusp according as the conic is a hyperbola, ellipse, or parabola. The factors of the terms of the fourth degree, viz: $(x + iy)(x + iy)(x - iy)(x - iy)$, show that the two imaginary circular points at infinity are double points on the quartic, which is thus trinodal. Hence this nodal, bicircular quartic can be projected into the most general form of the trinodal quartic. Trinodal quartics are unicursal.

If the conic which we invert be a parabola, the quartic has two nodes and one cusp. If the conic be inverted from a focus, the quartic has the two circular points at infinity for cusps. This is best shown analytically as follows: let the equation of the conic, origin being at the focus, be written

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{2aex}{a^2} - \frac{b^2}{a^2} = 0.$$

Inverting this we have

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{2aex(x^2 + y^2)}{a^2} - \frac{b^2(x^2 + y^2)^2}{a^2} = 0.$$

Now transform this equation so that the lines joining the origin to the circular points at infinity shall be the axes of reference. To do this

let $x + iy = x_1$ and $x - iy = y_1$;

$$\therefore x = \frac{x_1 + y_1}{2} \quad \text{and} \quad y = \frac{x_1 - y_1}{2i}$$

Making these substitutions and reducing we have (dropping the subscripts),

$$\frac{(x^2 + 2xy + y^2)}{a^2} - \frac{(x^2 - 2xy + y^2)}{b^2} - \frac{4aexy(x + y)}{a^2} - \frac{b^2x^2y^2}{a^2} = 0.$$

Making this equation homogeneous by means of z , we have

$$z^2 \frac{(x^2 + 2xy + y^2)}{a^2} - \frac{(x^2 - 2xy + y^2)}{b^2} - \frac{4aexy(x + y)}{a^2} - \frac{b^2x^2y^2}{a^2} = 0.$$

which is the equation of the quartic referred to the triangle formed by the three nodes. We are now able to determine the nature of the node at the vertex (y, z). Factor x^2 out of all the terms which contain it; and arrange thus:

$$\begin{aligned} x^2 \frac{z^2}{a^2} - \frac{z^2}{b^2} - \frac{4aexyz}{a^2} - \frac{b^2y^2}{a^2} \\ + 2x \frac{yz^2}{a^2} + \frac{yz^2}{b^2} - \frac{2aexy^2z}{a^2} \\ + \frac{y^2z^2}{a^2} - \frac{y^2z^2}{b^2} = 0. \end{aligned}$$

The quantity which multiplies x^2 represents the two tangents at the double point (y, z); but this quantity is a perfect square and hence we have a cusp. In this way the point (x, z) may be shown to be a cusp. Lastly, when a parabola is inverted from the focus, we obtain a tricuspidal quartic.

The trinodal quartic can be generated in a manner analogous to that shown for the nodal cubic. Let two projective pencils of rays have their vertices at A and B, the locus of intersection of corresponding rays is a conic through A and B. Invert from any point O in the plane, and we obtain two systems of co-axial circles, O A being the axis of one and O B of the other. The locus of intersection of corresponding circles is a bicircular quartic having a node at O. Projecting the whole figure we have the following theorem:—two projective systems of conics through O P Q A and O P Q B generate by their corresponding intersections a trinodal quartic having its nodes at O, P, and Q, and passing through A and B.

It is evident that the quartic generated in this way may have three nodes, one node and two cusps, two nodes and one cusp, or three cusps, depending upon the nature of the conic inverted and the centre of inversion. Making this the basis of classification we thus distinguish four varieties of unicursal quartics. To these must be added a fifth variety, viz: the quartic with a triple point. Each of these varieties will be considered separately.

The method of treating unicursal quartics given in this and the next four sections is in some respects similar to that suggested by Cayley in Salmon's Higher Plane Curves. But the method here sketched out is very different in its point of view and much wider in its application, yielding a multitude of new theorems not suggested by Cayley's method.

TRINODAL QUARTICS.

The quartic with three double points is a curve of the sixth class having four double tangents and six cusps (Salmon's H. P. C. Art. 243). Hence its reciprocal is of the sixth degree with four double points, six cusps, three double tangents, and no points of inflection.

The locus of intersection of tangents to a conic at right angles to one another is a circle. Inverting:—the locus of intersection of circles through the node and tangent a nodal, bicircular quartic and at right angles to one another is a circle. Projecting:—through the three nodes of a quartic draw two conics, each touching the quartic and intersecting so that the two tangents to the conics at their point of intersection, together with the lines from it to two of the nodes, form a harmonic pencil; the locus of all such intersections is a conic through these two nodes. Whenever the two tangents to the quartic from the third node, together with the lines from it to the other two nodes, form a harmonic pencil, this last conic breaks up into two right lines.

Any chord of a conic through O is cut harmonically by the conic and the polar of O. Inverting from O and projecting:—from one of the nodes of a trinodal quartic draw the two tangents to the quartic (not tangents at the node); draw the conic through these two points of contact and the three nodes; any line through the first mentioned node is cut harmonically by this conic, the quartic and the line joining the other two nodes.

If a triangle circumscribe a conic, the three lines from the angular points of the triangle to the points of contact of the opposite sides intersect in a point. Inverting and projecting:—through the three nodes of a quartic draw three conics touching the quartic; through the point of intersection of two of these conics, the point of contact of the third, and the three nodes draw a conic; three such conics can be drawn and they pass through a fixed point.

The eight points of contact of two conics with their four common tangents lie on a conic, which is the locus of a point, the pairs of tangents from which to the two given conics form a harmonic pencil. Inverting and projecting:—two connodal trinodal quartics have four common tangent conics through the three nodes; their eight points of contact lie on another connodal trinodal quartic; if from any point on the last quartic four conics be drawn through the nodes and tangent in pairs to the first quartics, any line through a node is cut harmonically by these four conics.

The eight common tangents to two conics at their common points all touch a conic. Inverting and projecting:—two connodal trinodal quartics intersect in four other points; eight conics can be drawn through the three nodes tangent to the quartics at these points of intersection; these eight conics all touch another connodal trinodal quartic.

A series of conics through four fixed points is cut by any transversal in a range of points in involution. Inverting and projecting:—a series of connodal trinodal quartics can be passed through four other fixed points; any conic through the three nodes cuts the series of quartics in pairs of points which determine at a node a pencil in involution. The conic touches two of the quartics and the lines to the points of contact are the foci of the pencil.

If the sides of two triangles touch a given conic, their six angular points will lie on another conic. Inverting and projecting:—if two groups of three conics each be passed through three nodes

and tangent to the quartic, their six points of intersection (three of each group) lie on another connodal trinodal quartic.

If the two triangles are inscribed in a conic, their six sides touch another conic. Inverting and projecting:—if two groups of three conics each be passed through the three nodes of a quartic so that the three points of intersection of each group lie on the quartic, these six conics all touch another connodal trinodal quartic.

A triangle is circumscribed about one conic, and two of its angular points are on a second conic; the locus of its third angular point is a conic.—Inverting and projecting:—if three conics be drawn through the three nodes of two connodal trinodal quartics so that they all touch one of the quartics and two of their points of intersection are on the other quartic, the locus of their third point of intersection is a connodal trinodal quartic.

A triangle is inscribed in one conic and two of its sides touch a second conic; the envelope of its third side is a conic. Inverting and projecting:—if three conics be drawn through the three nodes of two connodal trinodal quartics so that their three points of intersection lie on one of the quartics and two of them touch the other quartic, the envelope of the third conic is another connodal trinodal quartic.

The theorems of this section are stated in the most general terms and are still true when one or more of the nodes are changed into cusps. It is therefore not necessary to give separate theorems for the case of one cusp and two nodes.

NODAL BICUSPIDAL QUARTICS.

A quartic with one node and two cusps is a curve of the fourth class, having one double tangent and two points of inflection (see Salmon). Hence its reciprocal is also a nodal bicuspidal quartic, a fact of which frequent note will be made in this section.

The inverse of a conic with respect to a focus is a curve called Pascal's Limaçon. From the polar equation of a conic, the focus being the pole, it is evident that the polar equation of the limaçon may be written in the form:

$$r = \frac{e}{p} \cos x + \frac{1}{p};$$

where e and p are constants, being respectively the eccentricity and semi-latus rectum of the conic.

From the above equation it is readily seen that the curve may be traced by drawing from a fixed point O on a circle any number of chords and laying off a constant length on each of these lines, measured from the circumference of the circle. The point O is the node of the limaçon; and the fixed circle, which I shall call the base circle, is the inverse of the directrix of the conic. This is readily shown as follows:—the polar equation of the directrix is $r = p / (e \cos x)$. Hence the equation of its inverse is $r = (e \cos x) / p$, which is the equation of the base circle of the limaçon.

The envelope of circles on the focal radii of a conic as diameters is the auxiliary circle. Inverting:—the envelope of perpendiculars at the extremities of the nodal radii of a limaçon is a circle with its centre on the axis and having double contact with the limaçon. Projecting:—from any point on a nodal bicuspidal quartic draw lines to the three nodes and a fourth line forming with

them a harmonic pencil; the envelope of all such lines is a conic through the two cusps and having double contact with the quartic; the chord of contact passes through the node and cuts the line joining the cusps so that this point of intersection, the two cusps, and intersection of the double tangent with the cuspidal line form a harmonic range. Reciprocating:—on any tangent to a nodal bicircular quartic take the three points where it cuts the two inflectional tangents and the double tangent, and a fourth point forming with these a harmonic range; the locus of all such points is a conic touching the two inflectional tangents and having double contact with the quartic; the pole of the chord of contact is on the double tangent; join this last point to the intersection of the inflectional tangents and join the node with the same intersection; these four lines form a harmonious pencil.

If the tangent at any point P of a conic meet the directrix in Q , the line PQ will subtend a right angle at the focus O ; the circle POQ has PQ for a diameter and hence cuts the conic at P at right angles. Inverting:—from any point P on the limaçon draw OP to the node O ; draw OQ perpendicular to OP meeting the base circle in Q ; PQ is normal to the limaçon at P . Projecting:—from any point P on a nodal bicuspidal quartic draw lines to the three nodes and a fourth harmonic to these three; from O draw lines to the two cusps and a fourth harmonic to these two and the line OP ; the locus of the intersection of the fourth line of each pencil is a conic through the three nodes. Call this the basal conic of the quartic. Reciprocating:—on any given tangent to a nodal bicuspidal quartic take its points of intersection with the double tangent and the inflectional tangents, and a fourth point harmonic with these; on the double tangent take its points of intersection with the given tangent and the inflectional tangents, and a fourth point harmonic with these; the envelope of the lines joining the fourth point of these two ranges is a conic touching the double and inflectional tangents.

The locus of the foot of the perpendicular from the focus on the tangent to a conic is the auxiliary circle. Inverting:—draw a circle through the node tangent to a limaçon; draw the diameter OP of this circle; the locus of P is a circle having double contact with the limaçon, the axis being the chord of contact. Cor.; the locus of the centre of the tangent circle is also a circle. Projecting:—through the three nodes of a nodal bicuspidal quartic draw any conic touching the quartic; the locus of the pole with respect to this conic of the line joining the two cusps is a conic; draw the chord OP of the first conic through the node O and the pole of the line joining the two cusps; the locus of P is a conic through the cusps, having double contact with the quartic.

If chords of a conic subtend a constant angle at the focus, the tangents at the ends of the chords will meet on a fixed conic, and the chords will envelope another fixed conic; both these conics will have the same focus and directrix as the given conic. Inverting:—draw two nodal radii of a limaçon OP and OQ , making a given angle at O ; the envelope of the circle POQ is another limaçon; the locus of the intersection of circles through O tangent to the limaçon at P and Q is another limaçon. These two limaçons have the same node and base circle as the given one. Projecting:—through the node O of a nodal bicuspidal quartic draw a pencil of radii in involution; let OP and OQ be a conjugate pair of these nodal radii; the envelope of the conic through P , Q , and the three nodes, is another quartic of the same kind; also draw conics through the three nodes tangent to the quartic at P and Q ; the locus of their point of intersection is another quartic of the same kind. These three quartics all have the same node, cusps, and base conic.

Every focal chord of a conic is cut harmonically by the curve, the focus, and directrix. Inverting:—every nodal chord of a limaçon is bisected by the base circle. Projecting:—every nodal chord of a nodal bicuspidal quartic is cut harmonically by the quartic, the base conic, and the line joining the two cusps. Reciprocating:—from any point on the double tangent of a nodal bicuspidal

quartic draw the other two tangents to the quartic and a line to the intersection of the inflectional tangents; the fourth harmonic to these lines envelopes a conic.

Since the limaçon is symmetrical with respect to the axis, it follows that the two points of inflection are situated symmetrically with respect to the axis. Hence the line joining the two points of inflection is parallel to the double tangent. Therefore by projection we infer the following general theorem for the nodal bicuspidal quartic: the line joining the two cusps, the line joining the two points of inflection, and the double tangent meet in a point. Also the fourth harmonic points on each of these lines lie on a line through the node. Reciprocating:—the point of intersection of the cuspidal tangents, the point of intersection of inflectional tangents, and the node all lie on a right line. From the node draw a fourth harmonic to this right line and the tangents at the node; draw a fourth line harmonic to this right line and the inflectional tangents; draw a fourth harmonic to the cuspidal tangents and this right line; these three lines all meet in a point on the double tangent.

TRICUSPIDAL QUARTICS.

A tricuspidal quartic is a curve of the third class with one double tangent and no inflection. Its reciprocal is therefore a nodal cubic.

We shall begin by reciprocating some of the simpler properties of nodal cubics. Since the three points of inflection of a nodal cubic lie on a right line, it follows that the three cuspidal tangents of a tricuspidal quartic meet in a point. The reciprocal of the harmonic polar of a point of inflection is a point on the double tangent, found by drawing through the point of intersection of the three cuspidal tangents a line forming with them a harmonic pencil. Three such lines can be drawn and it is not difficult to distinguish them. All six lines form a pencil in involution, the lines to the points of contact of the double tangent being the foci. I shall call such a point on the double tangent the harmonic point of the cuspidal tangent. Since any two inflectional tangents of a nodal cubic meet on the harmonic polar of the third point of inflection, it follows that any two cusps of a trinodal quartic and the harmonic point of the third cuspidal tangent lie on a right line. Since the point of contact of the tangents from a point of inflection of a nodal cubic is on the harmonic polar of the point, it follows that the tangent to the tricuspidal quartic at the point where it is cut by a cuspidal tangent passes through the harmonic point of that cuspidal tangent.

The inverse of the parabola from a focus is the cardioid; and the inverse of the corresponding directrix is the base circle of the cardioid. The cardioid projects into a tricuspidal quartic and its base circle projects into a conic through the three cusps which has the same general properties as the base conic of the nodal bicuspidal quartic.

The circle circumscribing the triangle formed by the three tangents to a parabola passes through the focus. Inverting:—three circles through the cusp, and tangent to a cardioid, intersect in three collinear points. Projecting:—three conics through the three cusps of a tricuspidal quartic and touching the quartic intersect in three collinear points. Reciprocating:—if three conics touch the three inflectional tangents of a nodal cubic and the cubic itself, their three other common tangents intersect in a point.

Circles described on the focal radii of a parabola as diameters touch the tangent through the vertex. Inverting and projecting:—from a point on a tricuspidal quartic lines are drawn to the three cusps and a fourth line forming a harmonic pencil; the envelope of this fourth line is a conic through the three cusps and touching the quartic at the point where the latter is cut by one of the cuspidal tangents. There are three such conics, one corresponding to each cusp. At any cusp the tangent to its corresponding base conic, the cuspidal tangent, and the lines to the other two cusps

form a harmonic pencil. Reciprocating:—on any tangent to a nodal cubic take the three points of intersection with the inflectional tangents and a fourth point forming with these a harmonic range; the locus of this fourth point is a conic touching the three inflectional tangents and the cubic. The tangent to the cubic where it is touched by the conic goes through a point of inflection. On any inflectional tangent the point of contact of this conic, the point of inflection, and the points of intersection of the other two inflectional tangents form a harmonic range.

The circle described on any focal chord of a parabola as diameter will touch the directrix. Inverting:—the circle described on any cuspidal chord of a cardioid will touch the base circle. Projecting:—through a cusp C draw any chord of a tricuspidal quartic meeting the quartic in P and O ; draw a conic through P , O , and the other two cusps so that the pencil at P formed by the tangent to the conic and the lines to the cusps is harmonic; all such conics will touch the base conic of the cusp C . Reciprocating:—from O , on any inflectional tangent of a nodal cubic, draw two tangents P and Q to the cubic; draw a conic touching the tangents P and Q and the other two inflectional tangents so that the range on one of these tangents formed by the point of contact of the conic and the intersection of the three inflectional tangent is harmonic; the envelope of all such conics is a conic touching the three inflectional tangents.

The directrix of a parabola is the locus of the intersection of tangents at right angles to one another. Inverting and projecting:—through any point P on the base conic of a cusp C of the tricuspidal quartic, two conics can be drawn through the three cusps and touching the quartic; their two tangents at P and the lines to the other two cusps form a harmonic pencil; their two points of contact lie on a line through C . Reciprocating:—from any point on one of the inflectional tangents to a nodal cubic draw the two tangents P and Q ; draw two conics each touching the cubic and the three inflectional tangents, one touching P and the other Q ; the envelope of their other common tangent is a conic touching the three inflectional tangents; the two points of contact of any one of these common tangents and the points where it cuts the other two inflectional tangents form a harmonic range.

Any two parabolas which have a common focus and their axes in opposite directions cut at right angles. Inverting:—any two cardioids having a common cusp and their axes in opposite directions cut at right angles. Projecting:—two tricuspidal quartics having common cusps and at one of the cusps the same cuspidal tangent, but the cusps pointed in opposite directions, cut at such an angle that the tangents at a point of intersection and the lines to the other two cusps form a harmonic pencil. Reciprocating:—two nodal cubics have common inflectional tangents and on one of them the points of inflection common, but the branches of the curve on opposite sides of the line; any common tangent to the two curves is cut harmonically by the points of contact and the other two inflectional tangents.

Circles are described on any two focal chords of a parabola as diameters; their common chord goes through the vertex of the parabola. Inverting:—circles are described on any two cuspidal chords of a cardioid; the circle through their points of intersection and the cusp goes also through the vertex of the cardioid. Projecting:—through one of the cusps of a tricuspidal quartic draw two chords; draw conics through the other two cusps and the extremities of each of these chords so that the pole of the line joining the other two cusps with respect to each of these conics is on the corresponding chord; the conic through the points of intersection of these two conics and the cusps passes also through the point where the cuspidal tangent of the first mentioned cusp cuts the quartic. Reciprocating:—on one of the inflectional tangents, of a nodal cubic take two points P and Q ; draw a pair of tangents from each of these points to the cubic; draw two conics each touching a pair of these tangents and the other two inflectional tangents, so that the polars of the point of intersection of the other two inflectional tangents with respect to each of those conics pass

respectively through P and Q; the conic touching the common tangents to these two conics and the three inflectional tangents touches also the tangent from the first mentioned point of inflection to the cubic.

QUARTICS WITH A TRIPLE POINT.

Since a triple point is analytically equivalent to three double points, a quartic with a triple point is unicursal. Such a quartic is obtained by inverting a unicursal cubic from its node. The equation of such a cubic may be written

$$u_2 + u_3 = 0 ,$$

where u_2 and u_3 are homogeneous functions of the second and third degree respectively in x and y . Hence the equation of the inverse curve is

$$u_3 + u_2(x^2 + y^2) ,$$

which shows that the origin is a triple point and the quartic circular. By projecting this all other forms may be obtained.

The nature of the triple point depends upon the relation of the line at infinity to the cubic before inversion. Thus the line at infinity may cut the cubic in three distinct points all real, or one real and two imaginary, in one real and two coincident points (an ordinary tangent), or in three coincident points (an inflectional tangent). Hence the quartic may have at the triple point three distinct tangents all real, or one real and two imaginary, one real and two coincident, or all coincident.

This quartic may be generated in a manner similar to that used for the curves already discussed. We showed in the section on nodal cubics that a system of conics through A, B, C, D, and a projective pencil of rays with its vertex at A generate by the intersection of corresponding elements a cubic with a node at A. Invert the whole figure from A and then project:—the pencil of rays remains a pencil; the system of conics becomes a system of unicursal cubics having a common node at A and passing through five other common points; the cubic inverts and projects into a quartic with a triple point at A, passing through the five other common points of the system of cubics.

The three points of inflection of a nodal cubic lie on a right line. Inverting:—there are three points on a circular quartic with a triple point whose osculating circles pass through the triple point, and these three points lie on a circle through the triple point. Let these three points be designated by A, B, and C. The lines from the triple point O to the points A, B, C, and the common chord of the osculating circles at two of them form a harmonic pencil. Through one of these points, A, and the triple point draw a circle touching the quartic; the point of contact is on the common chord of the osculating circles at B and C.

From theorems which we have already proved for a system of cubics having a common node and passing through five others fixed points, we can infer other theorems for a system of quartics having a common triple point and passing through seven other fixed points. For example, any conic through the common double point and two of the fixed points is cut by the cubics in pairs of points which determine at the node a pencil in involution. Hence any cubic having its node at the common triple point and passing through any four of the fixed points is cut by the quartics in pairs of points which determine at the common triple point a pencil in involution. Again, the pairs of tangents to the cubics at the common double point form a pencil in involution, the two cuspidal tangents being

the foci of the pencil. Inverting:—the line at infinity (which passes through two of the fixed points, i. e. the circular points) cuts the system of circular quartics in pairs of points in involution. Projecting:—a line through any two of the seven fixed points cuts the system of quartics in pairs of points in involution. Since the line at infinity touches the inverse of a cuspidal cubic, it follows that any line through two of the fixed points will touch two of the quartics of the system; these points of contact are therefore the foci of the involution.

Other theorems on such a system of quartics will be given in the next section.

SYSTEMS OF QUARTICS THROUGH SIXTEEN POINTS.

Let U and V represent a system of quartics through sixteen points. Since the discriminant of quartic is of the 27th degree in the coefficients it follows that there are 27 values of k for which the discriminant vanishes, and hence 27 quartics of the system which have double points. As in case of cubics these 27 points are called the critic centres of the system. Let the equation of the system of quartics be written

$$u_4 + u_3 + u_2 + u_1 + u_0 = 0.$$

In a manner similar to that employed for cubics, we find the equation of the polar cubics of the origin with respect to the system to be

$$u_3 + 2u_2 + 3u_1 + 4u_0 = 0.$$

The polar conics of the origin are given by

$$u_2 + 3u_1 + 6u_0 = 0 ;$$

and the polar lines of the origin, by

$$u_1 + 4u_0 = 0 .$$

The origin may be any point in the plane and hence we conclude that only one quartic of the system passes through a given point and that the polar cubics of any point form a system through nine points. The polar conics of any point form a system through four points and the polar lines meet in a point.

If one of the critic centres be taken for origin, we can readily see that such a point is also a critic centre on each of its systems of polar curves. It is thus at a vertex of the self-polar triangle of its system of polar conics and the opposite side of the triangle is the common polar line of the critic centre with respect to each of the systems of curves. The tangents at the node of the nodal quartic coincide with those of its polar cubic and these we know coincide with the lines which constitute its polar conic.

If two of the sixteen basal points coincide, such a point is a critic centre. The argument is the same as for a system of cubics. We can also see that two of the basal points of each of its systems of polar curves coincide at the critic centre. The sixteen basal points of the system of quartics may unite two and two so that it is possible to draw a system of quartics touching eight given lines each at a fixed point.

If three of the basal points of our system of quartics coincide, all the quartics have at such a point a common point of inflection and a common inflectional tangent. The demonstration is the same as that already given for cubics. The system of polar cubics of such a point also have this point for a common point of inflection and the same tangent for a common inflectional tangent. I prefer to show this analytically for the sake of the method. The equation of the system of quartics having the origin for a common point of inflection and the axis of y for a common inflectional tangent may be written

$$u_4 + u_3 + \{ (B + kB_1)xy + (C + kC_1)y^2 \} + (A + kA_1)y = 0 .$$

The equation of the polar cubics of the origin is therefore,

$$u_3 + 2 \{ (B + kB_1)xy + (C + kC_1)y^2 \} + 3(A + kA_1)y = 0 ,$$

which proves the proposition. The properties of the system of polar conics of such a point are therefore the same as those already proved for cubics. One quartic of the system has a double point at the common point of inflection of the others.

When four basal points coincide they give rise either to a common point of undulation or a common double point on all the quartics of the system. The equation of the system having a common point of undulation may be written

$$u_4 + (A + kA_1)x^2y + (B + kB_1)xy^2 + (C + kC_1)y^3 \\ + (D + kD_1)xy + (E + kE_1)y^2 + (F + kF_1)y = 0 .$$

There is one value of k for which the last term vanishes, and hence the origin is a critic centre. The polar cubics of the point of undulation break up into a system of conics through four points and the common tangent at the common point of undulation. For the equation of the polar cubics is

$$y \{ (A + kA_1)x^2 + (B + kK_1)xy + (C + kC_1)y^2 \\ + 2(D + kD_1)x + 2(E + kE_1)y + (F + kF_1) \} = 0 .$$

The system of polar conics of the origin consequently breaks up into the line $y = 0$ and a pencil meeting in a point. The common tangent at the common point of undulation is also the common polar line of the point of undulation.

When the four coincident basal points form a common double point on the quartic, it is not difficult to show that two of the quartics are cuspidal at this point. The polar cubics of the common double point form a system having the same point for common double point. The tangents to the quartics at the common node constitute the system of polar conics and form a pencil in involution. Twelve of the sixteen basal points may unite in three groups of four each and the system of quartics is then trinodal and passes through four other fixed points. This is the system obtained by inverting a system of conics through four points and then projecting.

A few special cases should be noticed here. If the four fixed points and two of the nodes lie on a conic, this conic together with the two lines from the third node to the first two constitute a quartic of the system. If the four fixed points lie on a line, the quartic then consists of this line and the sides

of the triangle formed by the nodes. If the three nodes and three of the fixed points lie on a conic, the system of quartics then consists of this conic and a system of conics through the three nodes and the fourth fixed point. A special case of a system of quartics with three nodes is a system of cubics having a common node and passing through five other fixed points together with a line through two of them.

If a fifth basal point be moved up to join the four at the common node, the quartics have one tangent at the common node common to all. If six basal points coincide they have both tangents at the node common to all. In this case one of the quartics has a triple point at the common node of the others. If seven basal points coincide, one of these tangents is an inflectional tangent as well. If eight points coincide, both are inflectional tangents.

When nine of the basal points of a system of quartics coincide, the quartics have a common triple point. This is nicely shown by inverting a system of nodal cubics from the common node. The inverse curves form a system of quartics having a triple point and passing through seven other fixed points. The common triple point on two quartics counts for nine points of intersection and the seven others make the requisite sixteen. From our knowledge of a system of cubics having a common node it is readily inferred that three of the quartics must each break up into a nodal cubic and a right line through the node. If the seven fixed points of the system of quartics lie on a cubic having a node at the common triple point, the system of quartics then consists of this cubic and a pencil of lines through the node. If two of the seven fixed points lie on a line through the common triple point, the system of quartics then consists of this right line and a system of cubics through the other five points and having a common node at the common triple point.

The system of cubics having a common node may have one, two, or three of the other basal points at infinity; and these may be all distinct or two or three of them coincident. Whence we infer that if the system of quartics have ten coincident basal points, one of the tangents at the triple point is common to all the quartics of the system. If eleven basal points coincide, two of the triple-point tangents are common to all the quartics. If twelve coincide, all three triple-point tangents are common. These triple-point tangents may be all distinct, two coincident, or all three coincident.

If thirteen basal points coincide, the system of quartics then consists of the three fixed lines joining the multiple point to the other three, together with a pencil of lines through the multiple point. If fourteen points coincide, two lines are fixed and these with any two lines of the pencil form a quartic of the system. If fifteen points coincide, only one line is fixed and each quartic consists of this line and any other three of the pencil. When all sixteen points coincide, any four lines through it form a quartic of the system.

In this paper cubic and quartic curves only are considered. I expect in a future paper to extend the methods herein developed to curves of still higher degrees. Many of the present results can be generalized and stated for a unicursal curve of the n th degree. I have purposely omitted all consideration of focal properties of these curves. There are also many special forms of interest which do not properly belong to a general treatment of the subject.

NOTE A.

The theorem concerning the three points on a conic A, B, and C, whose osculating circles pass through a fourth point O on the conic, is due to Steiner. From the properties of the harmonic polars of the points of inflection on a nodal cubic we may infer many other theorems concerning the points A, B, and C on a conic. Let the cubic be projected into a circular cubic and then inverted from the node. Its points of inflection A_1 , B_1 , C_1 invert into the points A, B, and C. The harmonic

polar of A_1 inverts into the common chord $O P$ of the circles osculating the conic at B and C ; and similarly for the other harmonic polars.

The pencil $O \{A B P C\}$ is harmonic. Any circle through A and O meets the conic in S and T so that the pencil $O \{A S P T\}$ is harmonic. The two circles through O and tangent to the conic at S and T intersect on $O P$. If two circles be drawn through O and A intersecting the conic one in S and T and the other in U and V , the circles $O S U$ and $O T V$ intersect on $O P$; so also the circles $O S V$ and $O T U$. But one circle can be drawn through O and A and tangent to the conic; its point of contact is on $O P$. Let l, m , and n be three points on the conic on a circle through O . Draw the circles $O A l$, $O A m$, and $O A n$ intersecting the conic again in l_1, m_1, n_1 ; l_1, m_1, n_1 are also on a circle through O , and the circles through l, m, n and l_1, m_1, n_1 intersect on $O P$.

NOTE B.

From the fundamental property of the Cissoid of Diocles we can obtain by inversion an interesting theorem concerning the parabola. In the figure of the Cissoid given in Salmon's H. P. C. Art. 214, $AM_1 = MR$, whence $AM_1 = AR - AM$; or $AR = AM + AM_1$. Inverting from the cusp and representing the inverse points by the same letters, we have for the parabola

$$\frac{1}{AR} = \frac{1}{AM} + \frac{1}{AM_1}.$$

This result is interpreted as follows:—draw the circle of curvature at the vertex of a parabola; this circle is tangent to the ordinate BD which is equal to the abscissa AD ; draw a line through A cutting the circle in R , the ordinate BD in M , and the parabola in M_1 ; then

$$\frac{1}{AR} = \frac{1}{AM} + \frac{1}{AM_1}.$$

Draw the circle with centre at D and radius AD ; any chord of the parabola through the vertex is cut harmonically by the parabola, the circle, and the double ordinate through D .

Foreign Settlements in Kansas.

A CONTRIBUTION TO DIALECT STUDY IN THE STATE.

Explanatory.—Some years ago when the subject of dialect study in Kansas, or rather of Kansas dialect, was mentioned, Mr. Noble Prentis, a gentleman who is warranted in speaking with authority on Kansas, was inclined to think that he settled the question in short order by declaring that there is no Kansas dialect. Probably the majority of intelligent citizens of the state would turn off the subject with the same reply. In the sense of a mode of speech common to the inhabitants of Kansas and peculiar to them, Mr. Prentis was indeed right. There is no vocabulary, at least no extensive vocabulary, by which the native of Kansas may be recognized in the American Babel. We have no distinctive pronunciation by which we may be known from the inhabitants of Nebraska or set apart from the citizens of Missouri. The verb fails to agree with its subject and the participle is deprived of its final 'g' with about equal frequency in Western Kansas and Eastern Colorado.

But in the same sense it is true that there is no Kansas flora, no Kansas fauna; that is, there is no plant and there is no animal found quite generally in Kansas and found nowhere outside of Kansas. The remark that there is no such thing as a Kansas dialect rests upon a misapprehension of what is meant by the term. In just the same way that we speak of the flora and the fauna of Kansas we may speak of the dialect of Kansas. Yet to avoid popular misapprehension it may be better to speak of dialect in Kansas, rather than of Kansas dialect.

Dialect study involves the observation and description of all facts concerning the natural living speech of men, and especially those points in which the speech of individuals or groups differs from that of the standard literary language as represented in classic writers and classic speakers. Standard literary English is always a little behind the times. It is the stuffed and mounted specimen in the museum. Dialect is the live animal on its

native heath. Most people, indeed, will think that their speech does not differ materially from standard English. They say, "We speak near enough alike 'for practical purposes'. But a thousand years hence the pronunciation of our country may have changed so much that it will seem like another language, and our descendants will write learned theses to prove that we pronounced 'cough' like cow or like cuff. A new language will have grown out of an old one, but no one know how it came about. Careful dialect study will help explain it."

Kansas is a peculiarly favorable field for dialect study. We have here side by side representatives from nearly every state in the Union, and from a dozen foreign countries. The observer has here what elsewhere he must travel over half the world to find. In a district where the people are all natives, the speech is so nearly homogeneous that it is difficult to find any one who recognizes the peculiarities of his own language, but here the contrast of strange tongues strikes us immediately and we become conscious early of the fact that all men do not speak alike.

Study of dialect may be classified under the heads of pronunciation, grammar and vocabulary. Of these the last two are the easiest, and may be carried on by almost any one with pleasure and valuable results. Pronunciation is the most difficult of these matters to study, as competent observation and reports can be made only by one who has made a thorough study of Phonetics. To those who might wish to take up the study of this branch of the subject, Sweet's Primer of Phonetics, and Grandgent's "Vowel Measurements" and "German and English Sounds" are recommended.

In the study of dialect vocabularies it may become of the greatest importance to establish the exact locality of a word and the origin of the persons by whom it is used. For instance, in a family of my acquaintance the word 'slandering' = sauntering was familiar. It was a great puzzle to me until I learned that some of the children had been in the care of a German maid. The German word 'schlendern' suggested the unquestionable source of the peculiar word. As a source of information regarding the origin of the foreign elements of our population when their native speech shall have been forgotten, but when the influence of it will be left in vocabulary and pronunciation I have thought that a [map of the state](#) with the location of all the foreign settlements of even quite small size would be of interest and in time of great value. In the following pages I transmit the results of my

inquiries so far as received. It is my intention to make the report complete and to publish the map, when as complete as it can be made, in colors. Unexpected difficulties have delayed the work and prevented its being complete. I depended for my information upon the County Superintendents of the State, a class of unusually intelligent and well-informed men and women. But in not a few cases there seems to have been a suspicion in the mind of my correspondent that I might be a special officer of the state trying to locate violations of the law requiring district schools to be conducted in English, and hence information regarding schools in foreign tongue was withheld or given but partially. And in some cases my informants were not well posted. A superintendent by the name of Schauermann in a county containing a town called Suabia, tells me that there are no foreigners in his county. In such cases time must be taken to secure a correct result.

The questions asked were: Locate, and give origin, date and approximate numbers of any settlements—six or more families—of foreigners in your county. Do they still use their language to any extent? Do they have church service and schools conducted in their native tongue? In many replies one or more of these points was neglected so that the information is not yet by any means what I desire to make it. However, for the purpose of dialect study approximate correctness in location is of chief importance, and accuracy as to numbers quite secondary.

Through the aid of ministers and others to whom I have been referred by the superintendents I hope to make this report complete in the following respects: The more exact limits of the settlement; the numbers of those foreign-born; the province as well as land from which they came; the number of churches; the number of schools and the length of time the same are conducted. I solicit the co-operation of everyone interested in this work, and also in the whole subject of dialect study. As intimated above, interested observers can without especial training do a service to science and at the same time find a fascinating pastime for themselves by making collections of words and constructions which they believe to be unusual or new. If any such are sent to the writer they will be duly acknowledged. They should in every case be accompanied by a statement of the age, condition and birth-place of the person using them.

I wish here to call attention to the work of the American Dialect Society which exists to promote this study. It desires as wide a membership as possible, and membership is open to all interested in the subject. The publication of the Society, *Dialect Notes*, contains reports of word-lists and other studies, and will be an aid to any who wish to undertake similar work. Subscriptions and membership fees should be sent to Mr. C. H. Grandgent, Treas., Cambridge, Mass.

REPORTS BY COUNTIES.

ATCHISON.—Reports no foreigners, by John Klopfenstein, Supt.

ALLEN.—Swedes and Danes, from 600 to 700, settled from 1873 to 1880. Have church service, and four to five months school in Swedish. Grove and Elsmore townships. Germans in and around Humboldt.

ANDERSON.—Irish in Reeder township, 1860 and 1874. Germans, 1860 in Putman township, 1880 in Westphalia township. Have both church and schools in German.

BARBER.—Reports “no foreigners worth making account of”, by J. O. Hahn, Sup’t.

BARTON.—No report.

BOURBON.—Reports no foreigners.

BROWN.—No report.

BUTLER.—Germans (Prussians), speaking Low German, in Fairmount and Milton townships. Hold church services but no schools in German.

CHASE.—Russian Mennonites, speaking both Russian and German, in Diamond Creek township, no church, but a portion of

schooling in German. Germans at Strong City, with both church and schools in their native tongue.

CHATAUQUA.—Some Norwegians and Swedes, 1870, no location given. Neither schools nor churches in native tongue. One colony of 'Russians' (Mennonites?), who have also given up their language.

CHEROKEE.—Weir City, French and Italians, number considerable. Scammon, Scotch, also in large numbers. The French and Italians have neither schools nor church in the native tongue. Germans in Ross, twenty families; with church originally Lutheran, now Mennonite; school irregularly during past ten years. Swedes, a few families in Cherokee township, have entirely given up Swedish language. The Scotch, French and Italians in mines or mining industries.

CHEYENNE.—Germans settled in 1885-86 on Hackberry Creek, 160 persons; in the northeast corner of the county, 100; on west border of county, north of Republican river, 120; all with churches and the last two with occasional schools. Swedes are across the Republican adjoining last named German settlement, 120, entered 1886, having neither church nor school in Swedish.

CLARKE.—Reports no foreigners.

CLAY.—No report.

CLOUD.—Canadian French are scattered over much of the county, with considerable

settlements in and around the towns of Concordia, Clyde, St. Joseph and Aurora. In all there are churches, in the first three schools also conducted in French. Norwegians occupy portions of Sibley and Lincoln townships with two churches in their own tongue. They number about three hundred. Irish occupy portions of Solomon and Lyon, the south part of Meredith and the southeast corner of Grant townships.

COFFEY.—Germans in Liberty and on border of Leroy and Avon township. Have church service in German.

COMANCHE.—Germans. A few scattered families.

COWLEY.—A few Swedes and Germans, widely scattered.

CRAWFORD.—Irish and French in Grant township; Swedes in west part of Sherman township, have all given up their language. Italians, Austrians and other nationalities in south part of Washington, southeast part of Sheridan and all over Baker township, especially in Pittsburg, employed in mining and smelting.

DECATUR.—Swedes in Oberlin township; Mennonites in Prairie Dog township; Germans in and around Dresden, with Catholic church; Bohemians in Jennings and Garfield townships.

DICKINSON.—Germans, 500 in number settled in 1860 in Liberty, Union and Lyon townships. Have three churches and two

schools in German. Also in Wheatland, Jefferson, Bonner and Ridge townships, one church and a school. Swedes, 100 settled in 1860 in Center and Hayes township, with two churches and one school in Swedish. Irish, several hundred in south part of Banner township.

DONIPHAN.—Germans in Wayne, Marion and southern part of Center, Burr Oak and Washington townships, with church service in native tongue. Norwegians in eastern part of Wolf River township.

DOUGLAS.—There are German settlements in Eudora township (300), Marion township (600), and Big Springs township (100), with churches in all and school in the first. There are about five hundred Germans in Lawrence, with three German churches. There are smaller settlements of Germans and Scandinavians at several points in the county.

EDWARDS.—Germans and Swedes in Kinsley, Jackson and Trenton townships, have church service in their mother tongue.

ELK.—Swedes in Painter and Hood townships; Irish in Falls township; Germans on the border of Elk and Wild Cat townships. None of these have church or school in the native tongue, but all use it at home.

ELLIS.—Germans from Russia, settled about 1876 in Catherine, Hartsook, Lookout, Wheatland and Freedom townships, about 3000 in number—one third of the population of the county in 1891. They are Catholics, and have both churches and

parochial schools conducted in German.
They are large wheat-growers.

ELLSWORTH.—Germans, (Methodists) in south part of Valley township; Germans (Lutherans) in north part of Columbia and Ellsworth townships; Germans (Baptists) from Prussia, in Green Garden and south west corner of Empire townships. These all have church service, and the Lutherans schools in their own tongue. Bohemians in Valley and Noble townships.

FINNEY.—Reports no foreigners.

FORD.—Germans in Wheatland and Speareville townships. Have church, and one school conducted in German.

FRANKLIN.—No report.

GARFIELD.—A few scattered families of Germans.

GEARY.—Irish (Connaught) came into Jackson, Jefferson and Liberty townships 1855, about 1500 in number. Germans (Anhalt) about 1500 came into Jefferson, Milford and Lyon townships in 1862. About 300 English from Sussex settled in Lyon township in 1870. The Germans maintain both churches and schools in German.

GOVE.—Swedes in Lewis and south part of Grinnell and south west corner of Gove townships.

GRAHAM.—A settlement of Canadian French (600) was made in adjacent parts of Wild Horse and Morelan townships about 1880. They conduct church service but no schools in French.

GRANT.—Reports no foreigners.

GRAY.—Reports no foreigners.

GREELEY.—Swedes in the north west part of the county, have church service and summer school in Swedish.

GREENWOOD.—Norwegians, about 200, in south part of Salem township, have church in their own tongue. Germans in Shell Rock township, about 300, also have church in their own language.

HAMILTON.—Reports no foreigners.

HARPER.—Germans about the town of Harper. Hungarians south of Bluff City. Both have church service in German. About 100 French in Odell and Stohrville townships.

HARVEY.—Germans (Russian Mennonites) from Odessa, a few from Prussia, the latter speaking Low German. They settled from 1874 to 1876 in Alta and Garden townships, in Pleasant and the eastern part of Newton townships, and about Halstead. They have church and school in German, but speak Russian also. French in north part of Emma township, engaged in raising silk worms.

HASKELL.—Reports no foreigners.

HODGMAN.—Germans, about 30 families, settled about 1884 in south east corner of Sterling township; have preaching in German. Swedes in north west corner of Marena township.

JACKSON.—Danes in Netawaca and Whiting townships; Irish in Washington township;

neither continue to use their native tongue.

JEFFERSON.—Germans (Swiss) in Delaware, Jefferson and Kentucky townships, maintaining church but no schools in German.

JEWELL.—Swedes, widely scattered in Sinclair, Allen, Ewing and Ezbon townships.

JOHNSON.—No report.

KEARNEY.—No report.

KINGMAN.—A small settlement of Germans in Peters township, not using German to any extent. A few Irish in Union township.

KIOWA.—No report.

LABETTE.—Swedes and Norwegians settled in Valley and Canada townships about 1869. Still speak their language, but have neither church nor school in it.

LANE.—Reports no foreigners.

LEAVENWORTH.—German, in 1873 in Easton township; in Fair township in 1876; about 600 in each place. They have church service and schools in German.

LINCOLN.—Danes settled in Grant township in 1869 and since, 400 in number. Germans settled in Pleasant township in 1872, with 300, and in Indiana township in 1869 and later with about 375. Danes and Germans have good schools and churches in native tongue. Bohemians in Highland township in 1878 with thirty families. They speak their native tongue, but have no schools or churches.

LINN.—Reports no foreigners.

LOGAN.—Swedes, about 200, about Page City, in north part of county. Have church and school both in Swedish.

LYON.—Welsh, between 1000 and 1500 are located in and about Emporia, with three churches conducted in Welsh. There is a settlement of Scandinavians near Olpe in Centre township.

MARION.—Germans (Russian Mennonites), settled in Logan, Durham, Lehigh, Risley, Menno, West Branch and Liberty townships, from 1870 to 1875, some 5000 in number. They speak both Russian and German, and have church service and schools in the latter tongue. Bohemians, about 500 in number are settled in Clark township. They speak Czech and have church service in that language. French to the number of 200 settled soon after 1870 on the border of Grant and Doyle townships. They speak French still, but have no schools or church service in the language.

MARSHALL.—Germans (Pommeranians, Hanoverians, Frisians) to the number of 2000, settled in the west part of Marysville township from before 1860 to 1870. They have both church and school in their own tongue. In the north part of Murray and the south half of Vermillion townships are 1200 Irish, who use only English in church and school. They came before 1870. Bohemians in small numbers occupy the north part of Guittard, the north

west corner of Waterville and the south part of Blue Rapids townships; Swedes a portion of the south part of Waterville township. No report as to their language.

MEADE.—No report.

MIAMI.—Germans occupy the north part of Wea and the west part of Valley townships, about 200 in each settlement; the first has a Catholic, the second a Lutheran church. Irish occupy the north part of Osage township, also about 200 in number.

MITCHELL.—Germans to the number of 1200 occupy Pittsburg, Blue Hill, and Carr Creek townships. In the first there is a church, and a well-attended school (Catholic) at Tipton.

MONTGOMERY.—Germans to the number of 100 are settled in and about Independence. They have church service in German (Lutheran).

MORRIS.—Swedes occupy Diamond Valley, the west part of Creek, and the north part of Parker townships. They have several churches and occasionally a school conducted in Swedish.

MORTON.—Reports no foreigners.

MCPHERSON.—Swedes settled, about 1870, in Union, Smoky Hill, Harper, New Gottland, Delmore, and portions of other townships, in large numbers, several thousand. They have several churches and excellent schools conducted in Swedish. Germans (Russian Mennonites) occupy Superior, Turkey Creek, Mound, Lone

Tree, King City, and portions of McPherson and other southern townships, with several churches and schools. The Mennonites number about 5000 and settled after 1876.

NEMAHA.—Germans (Swiss) occupy Nemaha and Washington, and a portion of Richmond townships, with German churches and schools. Irish are in Clear Creek and north east corner of Neuchatel townships. Most of Neuchatel township is occupied by French (Swiss).

NEOSHO.—Germans have a considerable settlement in the south east corner of Tioga township, with church service (Lutheran) in German; another in the east part of Lincoln township, where the language is spoken, but without church or school. Swedes have settlements in the north west part of Tioga and the east part of Big Creek townships; church in the first only, though in both Swedish is spoken almost exclusively.

NESS.—No report.

NORTON.—Germans to the number of 100 settled about 1880 in Grant township. They have church service in German.

OSAGE.—Swedes, (700 in number,) settled in Grant township in 1871, where they have four churches conducted in Swedish. Welsh settled in 1869 in Arvon township, and others in the north part of Superior township, 700 in number. They have six churches with services in Welsh. Germans are in the north part of Scranton

and Ridgway townships, 200 in number; French in the central part of Superior township, 200 strong; Danes, 200, in north part of Melvern and Olivet townships; a considerable number of Scotch and Irish in and near Scranton. Most of these latter are engaged in coal mining. None of the foreigners have schools—except Sunday schools—in their native tongue.

OSBORNE.—Germans settled in Bloom township, where they have both church and school in their mother tongue.

OTTAWA.—Bohemians are located about the border of Sheridan and Fountain townships; Danes in the south part of Buckeye township; Irish, arrived about 1885, in the south part of Chapman township. None of these have church or school in a foreign tongue.

PAWNEE.—Swedes settled about 1877 in the west part of Garfield and the north part of Walnut townships, about 500 in all. They speak their native language at home almost exclusively, and have preaching in it.

PHILLIPS.—Germans occupy Mound and south part of Dayton townships, with preaching and parochial school in German. Dutch occupy east part of Prairie View with adjacent portions of Long Island, Dayton, and Beaver townships, with preaching in Dutch. Some Danes and Swedes in Crystal township, and some scattered Poles.

POTTAWATOMIE.—Germans, about 2500, in west half of Mill Creek and adjacent portions of Sherman and Vienna townships, also in Pottawatomie and adjacent portions of Union, Louisville, and St. George townships. There are a few families in Wamego and St. Mary's Mission. They have several schools and churches conducted in German. Swedes occupy the whole of Blue Valley and the west border of Greene townships, and have a small settlement in St. Mary township, numbering in all 1200. They have church service and a parochial school conducted in Swedish. Irish, to the number of 2000 occupy Clear Creek, Emmet, St. Mary and the border of St. Clere townships. French (Canadian), numbering 200, are found in the north part of Mill Creek and in Union townships, also a few about St. Mary's Mission.

PRATT.—Reports no foreigners.

RAWLINS.—Germans in north east part of county with church and school in German. Swedes in east part of county, Bohemians and Hungarians in north and north east portion.

RENO.—Germans, about 300, came in 1880 to north east corner of Little River township, and about 200 to south east corner of Sumner township; also a settlement in the west part of Hayes township; Dutch, about 350, came 1878 into Haven township; Russians are settled in Salt Creek and Medford townships. All have church service and schools in their native

tongue. There are also a few French and Danes in the county.

REPUBLIC.—No report.

RICE.—There is a considerable settlement of Germans in Valley township, also Pennsylvania Germans in the west part of Sterling township, with German churches in both. There are also some Germans in the town of Lyons, with a German church.

RILEY.—Swedes, about 2500, occupy Jackson, Swede Creek and adjacent portions of Mayday, Center, Fancy Creek and Sherman townships. They have church services and summer schools in their own tongue. Bohemians and Germans, about 500 together, occupy the north east part of Swede Creek township.

ROOKS.—Germans, 10 families, settled 1880 in north part of Northhampton township. Bohemians, 10 families, located in north part of Logan township in 1879. French, about 30 families, south west corner of Logan, and same number in Twin Mound township, settled in 1878, speak French and have preaching in that tongue. The Germans have church service in German.

RUSH.—Germans (Russian Mennonites) are located as follows: in Big Timber township 75 families, in Illinois township 25 families, in Pioneer township 50 families, in Lone Star township 50 families, in Banner township 25 families, in Garfield township 25 families, in Belle Prairie township 30 families. In each township there is one church or more, but

no German schools (?). Bohemians are found in Banner and Garfield townships, about 25 families in each.

RUSSELL.—No report.

SALINE.—Germans, (Bavarians and Swabians) about 200, in Gypsum and south part of Ohio townships; Swedes, 3000 to 4000, in Washington, Smolan, Falun, Liberty and Smoky View, and adjacent parts of Spring Creek, Summit and Walnut townships, also in Salina. The Swedes came in 1868. Both Germans and Swedes have preaching and the latter have schools in their tongue.

SCOTT.—No report.

SEDGWICK.—Germans, 3000 to 4000, settled from 1874-82 in Sherman, Grand River, Garden Plain, Attica and Union townships. Also about 2000 Germans in the city of Wichita. In both places schools and churches in German. Russians, Italians, French and Scandinavians are represented, a few hundred each, in Wichita. In the country townships a few Dutch and Swedes.

SEWARD.—Reports no foreigners.

SHAWNEE.—Germans (Moravians) in Rossville township, speak their native tongue almost exclusively, but have neither school nor preaching in German.

SHERIDAN.—No reports.

SHERMAN.—Germans, 20 families about the center of the county. Swedes, 10 families in north east corner and 25 families in

south west corner. Both Germans and Swedes have schools and preaching in their native tongue.

SMITH.—Germans in west part of Swan and Cedar townships, and on border of Harvey and Banner townships, in both churches, and in the first schools, in German. Dutch, in the south half of Lincoln township, have church but no schools.

STAFFORD.—Germans in Hayes and Cooper townships, three hundred in number, with two churches having service in German.

STANTON.—A few scattered Germans.

STEVENS.—No report.

SUMNER.—No report.

THOMAS.—A few foreigners scattered about the country; all anglicised.

TREGO.—No report.

WABAUNSEE.—Germans and some Swedes in Kaw, Newbury, Mill, Farmer, Alma and Washington townships, with both preaching and schools in the mother tongue.

WALLACE.—Swedes, to the number of 300, have settled since 1888 in the south west corner of the county. They have church and schools in Swedish.

WASHINGTON.—Germans in Franklin, Charleston, Hanover and north part of Sherman townships, have both church and schools (6) conducted in German. Bohemians are numerous in Little Blue township; French about midway in

Sherman township; Irish in Barnes, south part of Sherman and Koloko townships.

WICHITA.—No report.

WILSON.—Swedes have settled since 1870 in Colfax township. They have preaching but no schools in Swedish.

WOODSON.—No report.

WYANDOTTE.—Germans, 150, in north west corner of Prairie township; Swedes, 350, in Kansas City, Kas.; both have church service in the native language. Welsh, 200, in Rosedale, and Irish about midway in Wyandotte township.

SUMMARIES.

There are German settlements of thirty or more persons in the following counties: Allen, Anderson, Butler, Chase, Chautauqua, Cherokee, Cheyenne, Coffey, Comanche, Cowley, Crawford, Decatur, Dickinson, Doniphan, Douglas, Edwards, Elk, Ellis, Ellsworth, Ford, Garfield, Geary, Greenwood, Harper, Harvey, Hodgeman, Jefferson, Kingman, Leavenworth, Lincoln, Marion, Marshall, Miami, Mitchell, Montgomery, McPherson, Nemaha, Neosho, Norton, Osage, Osborne, Phillips, Pottawatomie, Rawlins, Reno, Rice, Riley, Rooks, Rush, Saline, Sedgwick, Shawnee, Sherman, Smith, Stafford, Stanton, Thomas, Wabaunsee, Washington, Wyandotte.

Total, 60.

Scandinavians in settlements of thirty or over are found in: Allen, Chautauqua, Cherokee, Cheyenne,

Cloud, Cowley, Crawford, Decatur, Dickinson, Doniphan, Edwards, Elk, Gove, Greeley, Greenwood, Hodgeman, Jackson, Jewell, Labette, Lincoln, Logan, Lyon, Marshall, Morris, McPherson, Neosho, Osage, Ottawa, Pawnee, Phillips, Pottawatomie, Rawlins, Riley, Saline, Sedgwick, Sherman, Wabaunsee, Wallace, Wilson, Wyandotte.

Total, 40.

Settlements of Slavonic peoples, Bohemians, Poles, Russians, or Hungarians, in: Decatur, Ellsworth, Harper, Lincoln, Marshall, Ottawa, Phillips, Rawlins, Reno, Riley, Rooks, Rush, Sedgwick, Washington.

Total, 14.

Settlements of Irish have been made in: Anderson, Cloud, Crawford, Dickinson, Doniphan, Elk, Geary, Jackson, Kingman, Marshall, Miami, Nemaha, Osage, Ottawa, Pottawatomie, Washington, Wyandotte.

Total, 17.

French are found in settlements of thirty or more in: Cherokee, Cloud, Crawford, Doniphan, Graham, Harper, Harvey, Nemaha, Osage, Pottawatomie, Rooks, Sedgwick, Washington.

Total, 13.

Italians are in Cherokee, Crawford, Sedgwick.

Total, 3.

Welsh in Lyon, Osage and Wyandotte.

Total, 3.

Dutch in Phillips, Reno, Sedgwick.

Total, 3.

Scotch are reported from Cherokee and Osage.

Total, 2.

English in Geary and Doniphan.

Total, 2.

The following counties report that there are no settlements of people of foreign birth within their borders: Atchison, Barber, Bourbon, Clarke, Finney, Grant, Gray, Hamilton, Haskell, Lane, Linn, Morton, Pratt, Seward.

Total, 14.

No reports have been secured from the following counties: Barton, Brown, Clay, Franklin, Johnson, Kearney, Kiowa, Meade, Ness, Republic, Russell, Scott, Sheridan, Stevens, Sumner, Trego, Wichita, Woodson.

Total, 18.

Seventy-four of our Kansas counties report settlements of citizens of foreign birth in numbers above 30. In so many cases there is no report or estimate of numbers that it is not worth while to give summaries. Probably there are not actually ten counties that have not such settlements.

Church services in a foreign tongue are held as follows: Allen S.,^[4] Anderson G., Butler G., Chase G., Cheyenne G., Cherokee G., Cloud F. S., Coffey G., Decatur G., Dickinson G. S., Doniphan G., Douglas G., Edwards G. S., Ellis G., Ellsworth G., Ford G., Geary G., Graham F., Greeley S.,

Greenwood G. S., Harper G. Hung., Harvey G.,
Hodgeman G., Jefferson G., Leavenworth G.,
Lincoln G. Du., Logan S., Lyon W. G., Marion G.
Boh., Marshall G., Miami G., Mitchell G.,
Montgomery G., Morris S., McPherson S. G.,
Nemaha G., Neosho G. S., Norton G., Osage S.
Welsh, Osborne G., Pawnee S., Phillips G. Du.,
Pottawatomie G. S., Rawlins G., Reno G. Du. Rus.,
Rice G., Riley S., Rooks F. G., Rush G., Saline G. S.,
Sedgwick G., Sherman G. S., Smith G. Du., Stafford
G., Wabaunsee G., Wallace S., Washington G.
Wilson S., Wyandotte G. S.

Total, 58.

This total of fifty-eight counties in which church service is held in a foreign tongue does not at all indicate the number of such churches. In many of the reports received the number is not given, or merely in the plural. These very incomplete reports indicate one hundred thirty-eight such churches; it is safe to say that the number is nearly double this.

More interesting is the number of schools conducted in a foreign tongue. The counties having them are: Allen S., Anderson G., Chase G., Cheyenne G., Cherokee, G., Cloud F., Dickinson G. S., Douglas G., Ellis G., Ellsworth G., Ford G., Geary G., Greeley S., Harvey G., Leavenworth G., Lincoln G. S., Logan S., Marion G., Marshall G., Mitchell G., Morris S., McPherson S. G., Nemaha G., Osborne G., Phillips G., Pottawatomie G. S., Rawlins G., Reno G. Du. Rus., Riley S., Rush G., Saline S., Sedgwick G., Sherman G. S., Smith G. Du., Wabaunsee G., Wallace S., Washington G.

Total, 37.

The number of separate schools in a foreign language so far as reported is seventy-four, and here, too, it is safe to say that the actual number is much larger.

EXPLANATION.

The spaces indicating settlements are in many cases too small to admit a complete description of the inhabitants, and accordingly they have been marked by races rather than by nationalities and tribes. "German" is made to do duty for all inhabitants of Germany whether Low or High, as well as for Austrians, German Swiss, and Russo-German Mennonites. The last are reported simply as Mennonites, but are, I believe, in all cases of German origin. "Scandinavian" is used instead of Swede, Norwegian and Dane, because in some cases the distinction was not made in the reports, and in order to limit the number of colors on the map which is to come. In the case of Scotch I have been unable to secure information whether they are Highlanders or Lowlanders, and in case of Irish, to what extent, if at all, they speak the old Irish language.

W. H. CARRUTH.

A PRELIMINARY MAP OF FOREIGN SETTLEMENTS IN KANSAS.

B - Bohemians (in a few cases other Slavs) G - Germans (including Dutch and Russian
Mennonites)

S - Scandinavians (Danes, Swedes and Norwegians) I - Irish W - Welsh It - Italians F - French
H - Hungarians

The Great Spirit Spring Mound.

BY E. H. S. BAILEY.

The "Waconda" or Great Spirit Spring, which is situated in Mitchell County, Kansas, about two miles east of Cawker City, has been described in detail by G. E. Patrick in vol. vii, p. 22, Transactions of the Kansas Academy of Science. An analysis of the water, and of the rock forming the mound on which the spring is located, is also given.

The spring is upon a conical, limestone mound 42 feet in height, and 150 feet in diameter at the top. The pool itself is a nearly circular lake about 50 feet in diameter, 35 feet deep, and the water rises to within a few inches of the top of the basin. There is a level space on all sides of the spring so wide that a carriage can be readily driven around it.

There is but little indication of organic matter in the water of the large spring, though there is a slimy white deposit adhering to the bottom and sides, but the water is colorless, clear, and transparent. The excess of water, instead of overflowing the bank, escapes by numerous small fissures, from 10 to 20 feet down on the sides, especially on the side away from the bluff. In these lateral springs there is an abundance of green algæ, and a whitish scum, which seems to be detached from the bottom and to float to the surface. This has a slimy, granular feeling suggesting in a very marked manner hydrated silica.

The mound is situated within about 200 feet of a limestone bluff, which rises perhaps 20 feet above the level of the spring. The natural inference would be that the harder material of the mound protected it from the erosion which carried away the rock in the valley of the Solomon on the south, and the rock between the spring and the bluff.

Is it not possible however that the mound has been really made by the successive deposits from the spring? Although the mound is plainly stratified, this need not interfere with the theory, for the water may have been intermittent in its flow. The rock is very porous, and on being ground to a thin section is shown to be concretionary in structure.

An analysis of the water of the spring (loc. cit.) showed that it contained over 1120 grains of mineral matter per gallon, of which 775 grains were sodium chloride and 206 grains sodium sulphate, with 66 grains of magnesium sulphate, 41 grains of magnesium carbonate, and 31 grains of calcium carbonate. An analysis by the author shows that there are 0.874 grains of silica.

Samples of the rock composing the mound, and of the adjoining bluff were secured, and comparative analyses made, with the following results:

	COUNTRY ROCK.	GREAT SPIRIT MOUND.
Silica and insoluble residue	2.14	4.10
Oxides of Iron and Alumina	[5] 3.22	2.66
Sulphuric Anhydride	.00	0.34
Carbon Dioxide	40.90	39.10
Calcium Oxide	51.90	49.28
Magnesium Oxide	0.63	1.15
Water and organic matter, undetermined	[6] <u>1.21</u>	<u>3.37</u>
	100.00	100.00
Specific gravity	2.52	2.79

The rocks are entirely different in appearance and structure, that of the mound being twice as hard as that of the bluff. The former contains much organic matter as is shown by blackening when it is heated in a tube and giving off the characteristic odor. The iron is practically of the ferrous variety, probably combined with carbonic acid, and the rock contains traces of chlorides. The particular sample taken was at some distance from the spring, and had been thoroughly exposed to the weather.

The rock of the mound is of just such a character as might have been built up by deposition from the water, as it contains the least soluble constituents of the water. The process of solidification would have been assisted by the silica in the water, forming insoluble cementing silicates, as

noticed by Prof. Patrick. The analysis given above shows that there is abundant silica in the water for this purpose.

Mention has been made of the organic growth in the adjacent springs. The mixed scum on being heated changes from a dull green to a vivid grass-green, and if ignited it swells up and emits an ill-smelling vapor, which is evidently nitrogenous in its character. A grayish white ash is left, which contains much carbonate of lime. This is evidently freshly deposited, as it is entangled in the algæ in granular lumps.

A specimen of the white scum, noticed above, only slightly mixed with the green algæ, was analyzed. The acid solution of the ash contains 1.26 per cent of soluble silica. This was of course a combined silica, probably calcium silicate, which becomes the cementing material in the rock. In another sample of ash, after removing all the substances soluble in hot water, the residue was found to contain 76.46 per cent of silica.

The siliceous residue from the scum was examined by Dr. S. W. Williston. It consists mostly of diatoms. He recognized

Navicula— 2 species
Nitzschia— 2 species
Asteronella— 1 species.

All three genera are found both in fresh and salt or brackish water.

The green material consists essentially of *Oscillaria* and *Confervæ*. If the scum is allowed to stand for a short time a very strong sulphuretted odor is developed, strangely suggestive of salt water marshes or mud flats; and indeed the same odor is noticed in the vicinity of the spring. No characteristic salt water organisms, that should occasion this peculiar odor have, however, yet been observed here. A more extended and special study of the organic life of these interior salt water marshes and springs would be of great interest.

On Pascal's Limaçon and the Cardioid.

BY H. C. RIGGS.

The inverse of a conic with respect to a focus is a curve called Pascal's Limaçon. From the polar equation of a conic, the focus being the pole, it is evident that the polar equation of the limaçon may be written in the form:

$$r = \frac{e}{p} \cos x + \frac{1}{p} ;$$

where e and p are constants, being respectively the eccentricity and semi-latus rectum of the conic.

From the above equation it is readily seen that the curve may be traced by drawing from a fixed point O on a circle any number of chords and laying off a constant length on each of these lines, measured from the circumference of the circle. The point O is the node of the limaçon; and the fixed circle, which I shall call the base circle, is the inverse of the directrix of the conic. This is readily shown as follows:—the polar equation of the directrix is $r = p / (e \cos x)$. Hence the equation of its inverse is $r = (e \cos x) / p$, which is the equation of the base circle of the limaçon.

If the conic which we invert be an ellipse, the point O will be an acnode on the Limaçon; if the conic be a hyperbola, the point O is a crunode. If the conic be a parabola, O is then a cusp and the inverse curve is called the Cardioid.

The limaçon may also be traced as a roulette.

Let the circle AC have a diameter just twice that of the circle AB . Then a given diameter of AC will always pass through a fixed point Q on the circle AB , (Williamson's Diff. Cal. Art. 286) and will have its middle point

on the circle AB . Now any point P on the diameter of AC will always be at a fixed distance from C and will therefore describe a limaçon of which AB will be the base circle.

The pedal of a circle with respect to any point is a limaçon. This may be inferred from the general theorem that the pedal of a curve is the inverse of its polar reciprocal, (Salmon's H. P. C. Art. 122). For the polar reciprocal of a conic from its focus is a circle and hence its pedal is a limaçon.

The base circle is the locus of the instantaneous centre for all points on the limaçon. Let BOQ be a line cutting a circle in B and Q . Let the line revolve about B, Q following the circle; the point P will trace a limaçon.

Now, for any instant, the instantaneous center will be the same whether Q be following the circle or the tangent at the point where the line cuts the circle. Therefore the instantaneous center for the point P is found by erecting a perpendicular to the line PB , through B , and a normal to the circle at Q . (Williamson's Diff. Cal. Art. 294). The intersection (C) of these two lines is the instantaneous center for the curve at the point P . But by elementary geometry C is on the circle. Now as the line PB revolves through 360° around B , the line BC which is always perpendicular to it also makes a complete revolution and the instantaneous center C moves once round the base circle.

Below we give a list of theorems obtained by inverting the corresponding theorems respecting a conic. In these theorems any circle through the pole is called a nodal circle, any chord through the pole is called a nodal chord, and the line through the pole perpendicular to the axis of the curve is called the latus rectum. The letters e and p signify respectively the eccentricity and half the latus rectum of the inverted conic.

The locus of the point of intersection of two tangents to a parabola which cut one another at a constant angle is a hyperbola having the same focus and same double point and director circle. The locus of the point of intersection of two nodal tangent circles to a cardioid which cut each other at a constant angle is a limaçon having the same focus and same double point and director circle. The locus of the point of intersection of two tangents to a parabola which cut one another at a constant angle is a hyperbola having the same focus and same double point and director circle. The locus of the point of intersection of two nodal tangent circles to a cardioid which cut each other at a constant angle is a limaçon having the same focus and same double point and director circle. The locus of the point of intersection of two tangents to a parabola which cut one another at a constant angle is a hyperbola having the same focus and same double point and director circle. The locus of the point of intersection of two nodal tangent circles to a cardioid which cut each other at a constant angle is a limaçon having the same focus and same double point and director circle.

The sum of the reciprocals of two focal chords of a conic at right angles to each other is constant. The sum of any two nodal chords of a limaçon at right angles to each other is constant.

P Q is a chord of a conic If P and Q be two points on a conic which subtends a right angle at the focus. The locus of the pole of P Q angle at the focus, then the locus of the point of intersection of the two nodal circles and the locus enveloped by P Q are each conics whose latera recta are to that of the original conic as $\sqrt{2} : 1$ and $1 : \sqrt{2}$ respectively. If P and Q be two points on a limaçon such that they intercept a right angle at the node, then the locus of the point of intersection of the two nodal circles and the locus enveloped by P Q are each conics whose latera recta are to that of the original limaçon as $\frac{1}{2}\sqrt{2} : 1$. And the envelope of the circle described on P Q as a diameter is a limaçon, whose latera recta are to that of the original limaçon as $1 : \frac{1}{2}\sqrt{2}$.

If two conics have a common focus, two of their common chords will pass through the point of intersection of their directrices. If two limaçons have a common node, two nodal circles passing each through two points of intersection of the limaçons, will pass through the point of intersection of their base circles.

Two conics have a common focus about which one of them is turned; two of their common chords will touch conics having the fixed focus for focus. Two limaçons have a common node about which one of them is turned; two of their common chords will envelope limaçons having fixed node for node.

Two conics are described having the same focus, and the distance of this focus from the corresponding directrix of each is the same; if the conics touch one another, then twice the sine of half the angle between the transverse axes of the conics is equal to the sine of half the angle between the transverse axes of the limaçons. If two limaçons are described having the same node and base circles of the same diameter, and if the limaçons touch each other, then twice the sine of half the angle between the transverse axes of the limaçons is equal to the sine of half the angle between the transverse axes of the conics.

axes is equal to the difference of the reciprocals of the eccentricities.

If a circle of a given radius pass through the focus (S) of a given conic and cut the conic in the points A, B, C, and D; then SA. SB. SC. SD is constant.

$$\frac{1}{(SA \cdot SB \cdot SC \cdot SD)} \text{ is constant.}$$

A circle passes through the focus of a conic whose latus rectum is 2l and meets the conic in four points whose distances from the focus are r_1, r_2, r_3, r_4 , then

$$r_1 + r_2 + r_3 + r_4 = 2l.$$

$$\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4} = \frac{2}{l}.$$

Two points P and Q are taken, one on each of two conics which have a common focus and their axes in the same direction, such that PS and QS are at right angles, S being the common focus. Then the nodal tangent circles at the tangents at P and Q meet on a circle whose square of whose eccentricity is equal to the sum of the squares of the eccentricities of the original conics.

A series of conics are described with a common latus rectum; the locus of points upon them at which the diameter

them at which the perpendicular of the nodal tangent circle is equal to from the focus on the tangent is the semi-latus rectum, is given by the equal to the semi-latus rectum is equation given by the equation

$$pr = -\cos 2x$$

$$p = -r \cos 2x$$

If POP_1 be a chord of a conic If POP_1 be a nodal circle of a through a fixed point O, then will limaçon passing through a fixed point $\tan \frac{1}{2}P_1SO \tan \frac{1}{2}PSO$ be a O, then will $\tan \frac{1}{2} P_1SO \tan \frac{1}{2} PSO$ be constant, S being the focus of the a constant, S being the node. conic.

Conics are described with Limaçons are described with equal equal latera recta and a common latera recta and a common node. Also focus. Also the corresponding the director circles envelop a fixed directrices envelop a fixed limaçon having a common node. Then confocal conic. Then these conics these limaçons all touch two fixed all touch two fixed conics, the limaçons whose latera recta are the sum reciprocals of whose latera recta and difference respectively of the are the sum and difference reciprocals of the variable limaçon and respectively of those of the of the fixed limaçon, and which have variable conic and their fixed the same base circle as the fixed confocal, and which have the same limaçon. directrix as the fixed confocal.

Every focal chord of a conic is Every nodal chord of a limaçon is cut harmonically by the curve, the bisected by the base circle. focus, and the directrix.

The envelope of circles on the The envelope of the perpendiculars focal radii of a conic as diameters at the extremities of the nodal radii of a is the auxiliary circle. limaçon is a circle having for the diameter the axis of the limaçon.

Below we give a number of theorems respecting the cardioid obtained by inverting the corresponding theorems concerning the parabola.

The straight line which bisects the angle contained by two lines drawn from the same point in a parabola, the one to the focus, the other perpendicular to the directrix, which is a tangent to the parabola at that point.	The nodal circle which bisects the angle between the line drawn from any point on a cardioid to the cusp and the nodal circle through the point perpendicular to the director circle is a tangent circle at that point.
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The latus rectum of a parabola is equal to four times the distance from the focus to the vertex.	The latus rectum of a cardioid is equal to its length on the axis.
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If a tangent to a parabola cut the axis produced, the points of contact and of intersection are equally distant from the focus.	If a nodal tangent circle cut the axis of a cardioid, the points of intersection and of tangency are equally distant from the cusp.
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If a perpendicular be drawn from the focus to any tangent to a parabola, the point of intersection will be on the vertical tangent.	If a nodal circle be drawn tangent to a cardioid, the diameter of such circle passing through the cusp will be a common chord of this circle and another described on the axis of the cardioid as diameter.
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The directrix of a parabola is the locus of the intersection of tangents that cut at right angles.	The base circle is the locus of the intersection of nodal circles tangent to a cardioid, which cut orthogonally.
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The circle described on any focal chord of a parabola as diameter will touch the directrix.	The circle described on any nodal chord of a cardioid as diameter will be tangent to the base circle.
--	--

The locus of a point from which two normals to a parabola can be drawn making complementary angles with the axis, is a parabola. The locus of the point through which two nodal circles, cutting a cardioid orthogonally, and making complementary angles with the axis, can be drawn is a cardioid.

Two tangents to a parabola which make equal angles with the axis and directrix respectively, but with the axis and latus rectum, are not at right angles, meet on the latus rectum. Two nodal circles tangent to a cardioid which make equal angles with the axis and latus rectum, orthogonally intersect on the latus rectum.

The circle which circumscribes the triangle formed by three tangents to a parabola passes through the focus. If three nodal circles be drawn tangent to a cardioid, the three points of intersection of these three circles are on a straight line.

If the two normals drawn to a parabola from a point P make equal angles with a straight line, the focus of P is a parabola. If the two nodal circles cutting a cardioid orthogonally and passing through the point P, make equal angles with a fixed nodal circle, the locus of P is a cardioid.

Any two parabolas which have a common focus and their axes in opposite directions intersect at right angles. Any two cardioids which have a common cusp and their axes in opposite directions intersect at right angles.

A number of other theorems on the limaçon and cardioid are given in Professor Newson's article in this number of the QUARTERLY, and these need not be repeated here.

Dialect Word-List.

BY W. H. CARRUTH.

The following are some of the dialect words that have come to one observer's ears within the past triennium. They are all from Kansas, unless otherwise noted. They are printed here to interest others, and to secure a basis for observation. The writer will be under obligations to any one who will note his familiarity with any of these words, insert others, or other meanings, and send them, with a statement of his place of birth and childhood, to him at Lawrence:

among: all of, as, Where are you going among you?

all: all gone, as, The corn is all. (Indiana, Penn.) Comp. German.

bat: a 'hard case.'

bid: in, to bid the time of day. (Indiana.)

beeslings: preparation of artificially curdled milk. (Indiana and Kansas.)

become: to look well in, as, He becomes that coat.

bad: desperate, as in, A bad citizen = a desperate fellow.

behave: to behave well, as in, Do behave now!

bump on a log: something lifeless, as, He sat there like a bump on a log.

bier: sham, as in pillow-bier. (Vermont.)

branch: a small stream.

breeze: a torrent of talk, as in, He gave me a breeze.

boo: dried mucous.

buckle down: to work persistently.

conniptions: a fit, also 'conniption fit.'

caba: an old valise. (Penn.)

craps: a game with dice; playing, it is called, 'shooting craps.'

crawl: to try to escape from an embarrassing situation without admitting one's mistake.

crawfish: same as 'crawl.'

crook: an earthenware vessel, a large bowl.

chuck: lunch.

chuck-a-luck: loaded (of dice).

coddy: odd, out of fashion.

chug: to strike a blow, as in, Chug him one.

could: to be able, as in, He used to could.

cod: a bit of deceit, as in, He gave the teacher a cod.

Chenuk: a Canadian. (Note the pronunciation.)

dast: to dare, as in, He don't dast to do it.

dew-claws: hands and knees (?), as in, Get down on your d.,
= apply yourself intensely.

Dick's hatband: in the phrase, As contrary as Dick's hatband.
What is the origin of this?

dick-nailer: anything quite satisfactory, as in,
He (it) is a dick-nailer.

drop: advantage, as in, to get the drop on a person (allusion to
dexterity in drawing a revolver). Comp. also: bulge, inside-
track,

whip handle, dead-wood, all used in the same way and with
same sense.

diven, past participle of *dive*.

drug: pret. of drag.

east: yeast.

emptings: bread dough set to ferment. Note the expression
"It will come out all right in the emptings," i. e. after it has had
a

chance to stand.

fat up: to increase a stake at cards.

find: to supply with board, as in, Pay five dollars a week and find
him;

I get five dollars and found.

fresh: impudent, (due to greenness).

fog: to filch.

fluke: to steal. (Indiana).

flat: plug tobacco. (Arkansas).

gallery: church, as in, He's in the gallery.

gag: an improbable story intended to deceive, as in, He tried to give me a gag.

go with: to become of, as in, What has gone with my hat? Ohio;
also in Pall Mall Gazette.

grub-stake: to give board.

girling: a 'girl-boy,' in contempt.

gaumy: not neat. (Arkansas.)

gob:, or **gaub:** a shapeless mass, as, a gob of mud, then sportively,
gaubs of wisdom.

gray: an awkward fellow.

get to: to get an opportunity to, as in, He didn't get to do it.

go to: to intend to, as in, I didn't go (for) to do it.

gumbo: a peculiar, putty-like dark soil. (Kansas.)

hen: feminine, as in, hen-party;
comp. stag-party, a gathering of men only.

honey: a fine fellow, generally ironically.

hump: to bestir, as, Hump yourself.

hole: bad condition financially; as, He is in the hole, i. e. he has lost.

huckleberry: indifferent, in, a huckleberry Christian.

huckleberry: the right person, as in, You're my huckleberry.

hornswoggle: to discomfit, as in, I'll be hornswoggled if I'll do it.

in it: on the successful side, as, He is not in it, i. e., He has no
prospect of success.

This phrase is universal in 1891.

infare: the reception after a wedding.

in: on the credit side, as, I was in five dollars.

jay: a green, conceited fellow.

jag: a bit of anything; a spree, a brief drunk.

jack mosquito: a large insect of the mosquito family,
three times the size of the pestiferous kind; this one does not
bite.

jimmy: to meddle, as, to jimmy with a thing or person,
to 'fool with.' Comp., to 'monkey with.'

jump: to leave without notice, as to jump the town,
to jump bail; to jump a board-bill is to leave it unpaid.

joint: an illegal saloon. What is the origin of it?

jigger or **chigger**: a minute red mite, which frequents weeds and lawns,

burrows beneath the human skin and causes excruciating itching.

keep: board and lodgings, as, He works for his keep.

lay over: to surpass, as, That lays over anything I know.

larofamedlers: a phrase used generally as equivalent to,

It's none of your business. (Maryland, Penn., Ohio, Arkansas.)

The word is a corruption of Lay-over for medlers, a lay-over being a bear-trap consisting of a pit covered with boughs.

light out: to start on the run, as, He 'lit' out for home.

lagniap: the extra in a bargain, as, Five dollars, and a hat for lagniap. (Louisiana.)

lush: to drink heavily, to 'swill.'

mog: to move, as, Mog along with you.

mogle: the same.

main: very, as, It's main strange. (Worcester county, Mass.)

mosey: to move along with a strut.

move: motion, as in, Get a move on you.

mealer: one who takes only meals at a boarding-house.

mind off: to ward off (flies, etc.).

meet up with: to meet. (Tennessee.)

peter out: to dwindle.

pail: to milk, as, to pail the cow. (Penn.)

possessed: anything, as, He acted like 'all possessed.'

quill: to write. (*The Writer*.)

quill-wheel: a 'rattle-trap' wagon.

ruther: choice, as, If I had my ruther; also, druther.

ride and tie: verbal phrase, describing a mode of travel in

which one vehicle is used by two sets of people, one riding ahead a

given distance and tying the team where the others who have walked

will come up to it, the first walking on ahead until overtaken and passed

by the second, and so on. (Colorado.)

red up: to make tidy.

ring off: to desist or cease talking, technical phrase from the telephone,

but passed into common usage. Comp., “saw off.”

rucus: quarrel, rumpus.

saddy: thanks, thank you. (Penn.)

saw off: ‘ring off;’ a short person is said to look ‘sawed off.’

shet, shut, shed: rid, as, to get shut of anything.

shear off: to pour off (water from settlings). (Ohio.)

shapin’s: young peas and beans—the unfilled pod. (Arkansas.)

should have said: said, as, He should have said yes,

i. e., indeed he said yes.

shin: to climb, as, to shin up a tree.

shut off: to make to stop talking, as, Do shut him off.

shebang: anything run-down, as house, carriage, affairs.

scrooch or scrooge: to cringe.

skin-away: a small boy. (Civilized Sac Indians.)

skin: to run, as, Skin out, i. e., run away.

skid: to sneak through examinations. (Yale.)

skid: a sharp-pointed instrument.

skit: a mild lie.

skads: great quantities, as, Skads of money, of books, etc.;

also = money, as, He hasn’t the skads.

singed cat: a shrewd ‘rustler,’ of unpretentious appearance.

skulduggery: knavery.

skip: to run away, as, Now skip, i. e., Go away from here.

skip: to leave hastily, as, He skipped the town.

slouch: a gawky fellow; then anything imperfect, as in the phrase,

He’s no slouch, i. e., He is an expert;

no slouch of a horse, i. e., a first-rate horse.

sloomiky: not neat.

slander: to saunter.

slump: to fail to meet requirements, as, in examinations.

slumps: great quantities. (Clark’s Second Hand Catalogue, N. Y.)

sleep: to give lodgings. I have heard, We can eat and sleep him.

smoodle: a sycophant. (Kansas University.) Comp., ‘swipe.’

smokewood: dried water-soaked wood used by small boys as substitute for cigars.

smearcase: a preparation of clabber, often called ‘Dutch cheese.’

snake: to snatch stealthily.

snum: to vow, as in, Well, I snum. Reported as common among girls.

snouge: unfair, as, a snouge game.

snide: inferior, unfair, as, a snide game, a snide watch, etc.

so fashion: thus, as, Do it so fashion.

soap: bribe money in elections.

sugar: same as soap.

sugar, (explet): pshaw!

split: anything, as, He ran like split, also, lickety split.

spunky: pouting, incensed.

sprinkle

sprinkling } : a small number; also a considerable number.

stag: masculine, as, A stag-party.

stag: to go to an entertainment without a lady companion,
as, to stag it.

Stoughton-bottle: an unimpressible fellow.

(From Stoughton’s Bitters, common in the 50’s.)

streak: rapid rate, as, He talked a streak,
or more commonly, a blue streak.

streak: to run, as, He streaked it for home.

steer: to manage (votes), as, A steering committee,
the same as ‘whips’ in Parliament.

striffin, or strifning: the membrane surrounding the abdominal viscera.
(Missouri.)

swan: to vow, as, in exclamation, I swan!

swat: to slap or strike, as, Swat him in the eye.

suz: (excl.): me, as in, Dear suz, and Law suz.

swipe: a sycophant. (Harvard.)

tacky: not fashionably dressed.

tewed: harrassed, as, I’m tewed and fretted.

that: so, as in, Not that far.

throw over: to ‘cut’ (an acquaintance).

throw over: to stop, as, I threw her over,
i. e., stopped talking. Common among railroad men; derived
from the use of the reverse lever.

tear-down: to thrash, as, He gave the boy a good tearing down.

toad on a tussock: anything dull or lifeless.

He sat there like a toad on a tussock.

tousle: to disarrange (hair).

tousey: frowsy.

topside: on top of, as in, The best man topside o' God's green earth.

trade-lash: an exchange of compliments. (Wellesley.)

trottin'-riggin's: best suit of clothes.

two sticks: anything, as, He's as cross as two sticks.

up above: up, as in, Up above stairs.

whootle-dasher: a 'rustler.'

want: for was not, were not, etc.

wamus, wampus, warmus: a close, generally knit jacket.

(Illinois, Pennsylvania, Ohio, Wisconsin, New England.)

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Footnotes:

[1] See note A.

[2] A few of the results of this section are due to the late Mr. H. B. Hall.

[3] See note B.

[4] G = German, S = Scandinavian, F = French, W = Welsh, Du = Dutch.

[5] Mostly FeO, and so calculated.

[6] With alkalies.

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