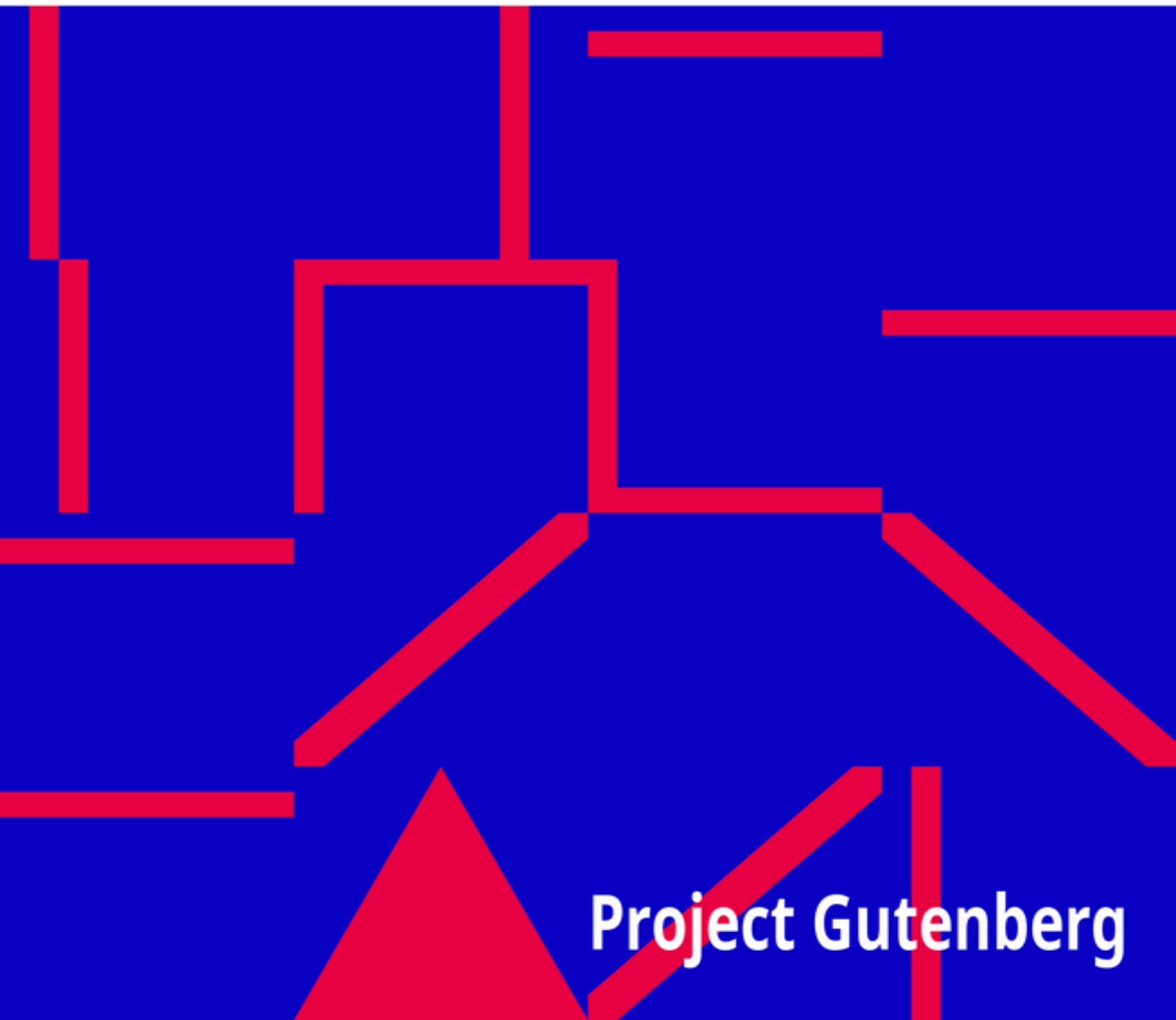


The golden mean

Jerry T. Bonnell and Robert J. Nemiroff

An abstract geometric pattern consisting of various red lines and shapes on a solid blue background. The pattern includes several horizontal and vertical lines of varying lengths, some forming right angles. There are also diagonal lines and a large, solid red triangle at the bottom left. The overall effect is a complex, layered geometric design.

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This is the golden ratio, $(1+\sqrt{5})/2$,

with 1.000.000 digits.

It is based on square root of 5 computed by Robert Nemiroff and Jerry Bonnell.

The golden ratio =

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