The Earliest Arithmetics in English

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This text includes characters that will only display in UTF-8 (Unicode) file encoding:

ȝ, ſ (yogh, long s)
ɳ, ɬɬ (n with curl, crossed l: see below)
φ (Greek phi, sometimes used in printed text for 0)

If any of these characters do not display properly, or if the apostrophes and quotation marks in this paragraph appear as garbage, you may have an incompatible browser or unavailable fonts. First, make sure that the browser's "character set" or "file encoding" is set to Unicode (UTF-8). You may also need to change your browser's default font.

In *The Crafte of Nombrynge*, final **n** was sometimes written with an extra curl as n with curl. It has been rendered as **ɳ** for visual effect; the character is not intended to convey phonetic information. In the same selection, the numeral "0" was sometimes printed as Greek φ (phi); this has been retained for the e-text. Double **l** with a line joined ll is shown as **ɬɬ**. The first few occurrences of **d** (for "pence") were printed with a curl as d with curl. The letter is shown with the same **d'** used in the remainder of the text.

The word "withdraw" or "w*ith*draw" was inconsistently hyphenated; it was left as printed, and line-end hyphens were retained. All brackets [ ] are in the original.

The diagrams in "Accomptynge by Counters" may not line up perfectly in all browsers, but the contents should still be intelligible.

The original text contained at least five types of marginal note. Details are given at the end of the e-text.

Typographical errors are shown in the text with mouse-hover popups. Other underlined words are cross-references to the Index of Technical Terms and the Glossary.

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(added by transcriber)

The Earliest Arithmetics in English

EDITED WITH INTRODUCTION

BY

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INTRODUCTION

THE number of English arithmetics before the sixteenth century is very small. This is hardly to be wondered at, as no one requiring to use even the simplest operations of the art up to the middle of the fifteenth century was likely to be ignorant of Latin, in which language there were several treatises in a considerable number of manuscripts, as shown by the quantity of them still in existence. Until modern commerce was fairly well established, few persons required more arithmetic than addition and subtraction, and even in the thirteenth century, scientific treatises addressed to advanced students contemplated the likelihood of their not being able to do simple division. On the other hand, the study of astronomy necessitated, from its earliest days as a science, considerable skill and accuracy in computation, not only in the calculation of astronomical tables but in their use, a knowledge of which latter was fairly common from the thirteenth to the sixteenth centuries.

The arithmetics in English known to me are:—

(1) Bodl. 790 G. VII. (2653) f. 146-154 (15th c.) *inc.* "Of angrym ther be IX figures in numbray . . ." A mere unfinished fragment, only getting as far as Duplation.

(2) Camb. Univ. LI. IV. 14 (III.) f. 121-142 (15th c.) *inc.* "Al maner of thyngis that prosedeth ffro the frist begynnyng . . ."

(3) Fragmentary passages or diagrams in Sloane 213 f. 120-3 (a fourteenth-century counting board), Egerton 2852 f. 5-13, Harl. 218 f. 147 and

(4) The two MSS. here printed; Eg. 2622 f. 136 and Ashmole 396 f. 48. All of these, as the language shows, are of the fifteenth century.

The CRAFTE OF NOMBRYNGE is one of a large number of scientific treatises, mostly in Latin, bound up together as Egerton MS. 2622 in the British Museum Library. It measures 7" × 5", 29-30 lines to the page, in a rough hand. The English is N.E. Midland in dialect. It is a translation and amplification of one of the numerous glosses on the *de algorismo* of Alexander de Villa Dei (c. 1220), such as that of Thomas of Newmarket contained in the British Museum MS. Reg. 12, E. 1. A fragment of another translation of the same gloss was printed by Halliwell in his *Rara Mathematica* (1835) p. 29.[1] It corresponds, as far as p. 71, l. 2, roughly to p. 3 of our version, and from thence to the end p. 2, ll. 16-40.

The ART OF NOMBRYNG is one of the treatises bound up in the Bodleian MS. Ashmole 396. It measures 11½" × 17¾", and is written with thirty-three lines to the page in a fifteenth century hand. It is a translation, rather literal, with amplifications of the *de arte numerandi* attributed to John of Holywood (Sacrobosco) and the translator had obviously a poor MS. before him. The *de arte numerandi* was printed in 1488, 1490 (*s.n.*), 1501, 1503, 1510, 1517, 1521, 1522, 1523, 1582, and by Halliwell separately and in his two editions of *Rara Mathematica*, 1839 and 1841, and reprinted by Curze in 1897.

Both these tracts are here printed for the first time, but the first having been circulated in proof a number of years ago, in an endeavour to discover other manuscripts or parts of manuscripts of it, Dr. David Eugene Smith, misunderstanding the position, printed some pages in a curious transcript with four facsimiles in the *Archiv für die Geschichte der Naturwissenschaften und der Technik*, 1909, and invited the scientific world to take up the "not unpleasant task" of editing it.

ACCOMPTYNGE BY COUNTERS is reprinted from the 1543 edition of Robert Record's Arithmetic, printed by R. Wolfe. It has been reprinted within the last few years by Mr. F. P. Barnard, in his work on Casting Counters. It is the earliest English treatise we have on this variety of the Abacus (there are Latin ones of the end of the fifteenth century), but there is little doubt in my mind that this method of performing the simple operations of arithmetic is

much older than any of the pen methods. At the end of the treatise there follows a note on merchants' and auditors' ways of setting down sums, and lastly, a system of digital numeration which seems of great antiquity and almost world-wide extension.

After the fragment already referred to, I print as an appendix the 'Carmen de Algorismo' of Alexander de Villa Dei in an enlarged and corrected form. It was printed for the first time by Halliwell in *Rara Mathemathica*, but I have added a number of stanzas from various manuscripts, selecting various readings on the principle that the verses were made to scan, aided by the advice of my friend Mr. Vernon Rendall, who is not responsible for the few doubtful lines I have conserved. This poem is at the base of all other treatises on the subject in medieval times, but I am unable to indicate its sources.

The Subject Matter.

Ancient and medieval writers observed a distinction between the Science and the Art of Arithmetic. The classical treatises on the subject, those of Euclid among the Greeks and Boethius among the Latins, are devoted to the Science of Arithmetic, but it is obvious that coeval with practical Astronomy the Art of Calculation must have existed and have made considerable progress. If early treatises on this art existed at all they must, almost of necessity, have been in Greek, which was the language of science for the Romans as long as Latin civilisation existed. But in their absence it is safe to say that no involved operations were or could have been carried out by means of the alphabetic notation of the Greeks and Romans. Specimen sums have indeed been constructed by moderns which show its possibility, but it is absurd to think that men of science, acquainted with Egyptian methods and in possession of the abacus, [2] were unable to devise methods for its use.

The Pre-Medieval Instruments Used in Calculation.

The following are known:—

(1) A flat polished surface or tablets, strewn with sand, on which figures were inscribed with a stylus.

(2) A polished tablet divided longitudinally into nine columns (or more) grouped in threes, with which counters were used, either plain or marked with signs denoting the nine numerals, etc.

(3) Tablets or boxes containing nine grooves or wires, in or on which ran beads.

(4) Tablets on which nine (or more) horizontal lines were marked, each third being marked off.

The only Greek counting board we have is of the fourth class and was discovered at Salamis. It was engraved on a block of marble, and measures 5 feet by 2½. Its chief part consists of eleven parallel lines, the 3rd, 6th, and 9th being marked with a cross. Another section consists of five parallel lines, and there are three rows of arithmetical symbols. This board could only have been used with counters (*calculi*), preferably unmarked, as in our treatise of *Accomptynge by Counters*.

Classical Roman Methods of Calculation.

We have proof of two methods of calculation in ancient Rome, one by the first method, in which the surface of sand was divided into columns by a stylus or the hand. Counters (*calculi*, or *lapilli*), which were kept in boxes (*loculi*), were used in calculation, as we learn from Horace's schoolboys (Sat. 1. vi. 74). For the sand see Persius I. 131, "Nec qui abaco numeros et secto in pulvere metas scit risisse," Apul. Apolog. 16 (pulvisculo), Mart. Capella, lib. vii. 3, 4, etc. Cicero says of an expert calculator "eruditum attigisse pulverem," (de nat. Deorum, ii. 18). Tertullian calls a teacher of arithmetic "primus numerorum arenarius" (de Pallio, *in fine*). The counters were made of various materials, ivory principally, "Adeo nulla uncia nobis est eboris, etc." (Juv. XI. 131), sometimes of precious metals, "Pro calculis albis et nigris aureos argenteosque habebat denarios" (Pet. Arb. Satyricon, 33).

There are, however, still in existence four Roman counting boards of a kind which does not appear to come into literature. A typical one is of the third class. It consists of a number of transverse wires, broken at the middle. On the left hand portion four beads are strung, on the right one (or two). The left hand beads signify units, the right hand one five units. Thus any number up to nine can be represented. This instrument is in all essentials the same as the Swanpan or Abacus in use throughout the Far East. The Russian stchota in use throughout Eastern Europe is simpler still. The method of using this system is exactly the same as that of *Accomptynge by Counters*, the right-hand five bead replacing the counter between the lines.

The Boethian Abacus.

Between classical times and the tenth century we have little or no guidance as to the art of calculation. Boethius (fifth century), at the end of lib. II. of his *Geometria* gives us a figure of an abacus of the second class with a set of counters arranged within it. It has, however, been contended with great probability that the whole passage is a tenth century interpolation. As no rules are given for its use, the chief value of the figure is that it gives the signs of the nine numbers, known as the Boethian "apices" or "notae" (from whence our word "notation"). To these we shall return later on.

The Abacists.

It would seem probable that writers on the calendar like Bede (A.D. 721) and Helpericus (A.D. 903) were able to perform simple calculations; though we are unable to guess their methods, and for the most part they were dependent on tables taken from Greek sources. We have no early medieval treatises on arithmetic, till towards the end of the tenth century we find a revival of the study of science, centring for us round the name of Gerbert, who became Pope as Sylvester II. in 999. His treatise on the use of the Abacus was written (c. 980) to a friend Constantine, and was first printed among the works of Bede in the Basle (1563) edition of his works, I. 159, in a somewhat enlarged form. Another tenth century treatise is that of Abbo of Fleury (c. 988), preserved in several manuscripts. Very few treatises on the

use of the Abacus can be certainly ascribed to the eleventh century, but from the beginning of the twelfth century their numbers increase rapidly, to judge by those that have been preserved.

The Abacists used a permanent board usually divided into twelve columns; the columns were grouped in threes, each column being called an “arcus,” and the value of a figure in it represented a tenth of what it would have in the column to the left, as in our arithmetic of position. With this board counters or jetons were used, either plain or, more probably, marked with numerical signs, which with the early Abacists were the “apices,” though counters from classical times were sometimes marked on one side with the digital signs, on the other with Roman numerals. Two ivory discs of this kind from the Hamilton collection may be seen at the British Museum. Gerbert is said by Richer to have made for the purpose of computation a thousand counters of horn; the usual number of a set of counters in the sixteenth and seventeenth centuries was a hundred.

Treatises on the Abacus usually consist of chapters on Numeration explaining the notation, and on the rules for Multiplication and Division. Addition, as far as it required any rules, came naturally under Multiplication, while Subtraction was involved in the process of Division. These rules were all that were needed in Western Europe in centuries when commerce hardly existed, and astronomy was unpractised, and even they were only required in the preparation of the calendar and the assignments of the royal exchequer. In England, for example, when the hide developed from the normal holding of a household into the unit of taxation, the calculation of the geldage in each shire required a sum in division; as we know from the fact that one of the Abacists proposes the sum: “If 200 marks are levied on the county of Essex, which contains according to Hugh of Bocland 2500 hides, how much does each hide pay?” [3] Exchequer methods up to the sixteenth century were founded on the abacus, though when we have details later on, a different and simpler form was used.

The great difficulty of the early Abacists, owing to the absence of a figure representing zero, was to place their results and operations in the proper columns of the abacus, especially when doing a division sum. The chief differences noticeable in their works are in the methods for this rule.

Division was either done directly or by means of differences between the divisor and the next higher multiple of ten to the divisor. Later Abacists made a distinction between "iron" and "golden" methods of division. The following are examples taken from a twelfth century treatise. In following the operations it must be remembered that a figure asterisked represents a counter taken from the board. A zero is obviously not needed, and the result may be written down in words.

(*a*) MULTIPLICATION. 4600 × 23.

Thousands						
Hundreds	Tens	Units	Hundreds	Tens	Units	
		4	6			**Multiplicand.**
		1	8			600 × 3.
	1	2				4000 × 3.
	1	2				600 × 20.
	8					4000 × 20.
1		5	8			Total product.
				2	3	**Multiplier.**

(*b*) DIVISION: DIRECT. 100,000 ÷ 20,023. Here each counter in turn is a separate divisor.

H.	T.	U.	H.	T.	U.	
	2			2	3	**Divisors.**
	2					Place greatest divisor to right of dividend.
1						**Dividend.**
	2					Remainder.
			1			
	1	9	9			Another form of same.

				8		Product of 1st Quotient and 20.
	1	9	9	2		Remainder.
				1	2	Product of 1st Quotient and 3.
	1	9	9		8	**Final remainder.**
					4	Quotient.

(*c*) DIVISION BY DIFFERENCES. 900 ÷ 8. Here we divide by (10-2).

			H.	T.	U.	
					2	Difference.
					8	Divisor.
			[4]9			**Dividend.**
			[4]1	8		Product of difference by 1st Quotient (9).
				2		Product of difference by 2nd Quotient (1).
			[4]1			Sum of 8 and 2.
				2		Product of difference by 3rd Quotient (1).
					4	Product of difference by 4th Quot. (2). **Remainder.**
					2	4th Quotient.
				1		3rd Quotient.
				1		2nd Quotient.
				9		1st Quotient.
			1	1	2	**Quotient.** (**Total of all four.**)

DIVISION. 7800 ÷ 166.

Thousands						
H.	T.	U.	H.	T.	U.	
				3	4	Differences (making 200 trial divisor).
			1	6	6	Divisors.
		[4]7	8			**Dividends.**
		1				Remainder of greatest dividend.
			1	2		Product of 1st difference (4) by 1st Quotient (3).
			9			Product of 2nd difference (3) by 1st Quotient (3).
		[4]2	8	2		New dividends.

			3	4		Product of 1st and 2nd difference by 2nd Quotient (1).
		$^{4}1$	1	6		New dividends.
				2		Product of 1st difference by 3rd Quotient (5).
			1	5		Product of 2nd difference by 3rd Quotient (5).
			$^{4}3$	3		New dividends.
			1			Remainder of greatest dividend.
				3	4	Product of 1st and 2nd difference by 4th Quotient (1).
			1	6	4	**Remainder** (less than divisor).
					1	4th Quotient.
					5	3rd Quotient.
				1		2nd Quotient.
				3		1st Quotient.
				4	6	**Quotient.**

Division. 8000 ÷ 606.

Thousands						
H.	T.	U.	H.	T.	U.	
				9		Difference (making 700 trial divisor).
					4	Difference.
			6		6	Divisors.
		$^{4}8$				**Dividend.**
		1				Remainder of dividend.
			9	4		Product of difference 1 and 2 with 1st Quotient (1).
		$^{4}1$	9	4		New dividends.
			3			Remainder of greatest dividend.
				9	4	Product of difference 1 and 2 with 2nd Quotient (1).
		$^{4}1$	3	3	4	New dividends.
			3			Remainder of greatest dividend.
				9	4	Product of difference 1 and 2 with 3rd Quotient (1).
			7	2	8	New dividends.
			6		6	Product of divisors by 4th Quotient (1).
			1	2	2	**Remainder.**
					1	4th Quotient.

					1	3rd Quotient.
					1	2nd Quotient.
				1		1st Quotient.
				1	3	**Quotient.**

The chief Abacists are Gerbert (tenth century), Abbo, and Hermannus Contractus (1054), who are credited with the revival of the art, Bernelinus, Gerland, and Radulphus of Laon (twelfth century). We know as English Abacists, Robert, bishop of Hereford, 1095, "abacum et lunarem compotum et celestium cursum astrorum rimatus," Turchillus Compotista (Thurkil), and through him of Guilielmus R. . . . "the best of living computers," Gislebert, and Simonus de Rotellis (Simon of the Rolls). They flourished most probably in the first quarter of the twelfth century, as Thurkil's treatise deals also with fractions. Walcher of Durham, Thomas of York, and Samson of Worcester are also known as Abacists.

Finally, the term Abacists came to be applied to computers by manual arithmetic. A MS. Algorithm of the thirteenth century (Sl. 3281, f. 6, b), contains the following passage: "Est et alius modus secundum operatores sive practicos, quorum unus appellatur Abacus; et modus ejus est in computando per digitos et junctura manuum, et iste utitur ultra Alpes."

In a composite treatise containing tracts written A.D. 1157 and 1208, on the calendar, the abacus, the manual calendar and the manual abacus, we have a number of the methods preserved. As an example we give the rule for multiplication (Claud. A. IV., f. 54 vo). "Si numerus multiplicat alium numerum auferatur differentia majoris a minore, et per residuum multiplicetur articulus, et una differentia per aliam, et summa proveniet." Example, 8 × 7. The difference of 8 is 2, of 7 is 3, the next article being 10; 7 - 2 is 5. 5 × 10 = 50; 2 × 3 = 6. 50 + 6 = 56 answer. The rule will hold in such cases as 17 × 15 where the article next higher is the same for both, *i.e.*, 20; but in such a case as 17 × 9 the difference for each number must be taken from the higher article, *i.e.*, the difference of 9 will be 11.

THE ALGORISTS.

Algorism (augrim, augrym, algram, agram, algorithm), owes its name to the accident that the first arithmetical treatise translated from the Arabic happened to be one written by Al-Khowarazmi in the early ninth century, "de numeris Indorum," beginning in its Latin form "Dixit Algorismi. . . ." The translation, of which only one MS. is known, was made about 1120 by Adelard of Bath, who also wrote on the Abacus and translated with a commentary Euclid from the Arabic. It is probable that another version was made by Gerard of Cremona (1114-1187); the number of important works that were not translated more than once from the Arabic decreases every year with our knowledge of medieval texts. A few lines of this translation, as copied by Halliwell, are given on p. 72, note 2. Another translation still seems to have been made by Johannes Hispalensis.

Algorism is distinguished from Abacist computation by recognising seven rules, Addition, Subtraction, Duplation, Mediation, Multiplication, Division, and Extraction of Roots, to which were afterwards added Numeration and Progression. It is further distinguished by the use of the zero, which enabled the computer to dispense with the columns of the Abacus. It obviously employs a board with fine sand or wax, and later, as a substitute, paper or parchment; slate and pencil were also used in the fourteenth century, how much earlier is unknown. [5] Algorism quickly ousted the Abacus methods for all intricate calculations, being simpler and more easily checked: in fact, the astronomical revival of the twelfth and thirteenth centuries would have been impossible without its aid.

The number of Latin Algorisms still in manuscript is comparatively large, but we are here only concerned with two—an Algorism in prose attributed to Sacrobosco (John of Holywood) in the colophon of a Paris manuscript, though this attribution is no longer regarded as conclusive, and another in verse, most probably by Alexander de Villedieu (Villa Dei). Alexander, who died in 1240, was teaching in Paris in 1209. His verse treatise on the Calendar is dated 1200, and it is to that period that his Algorism may be attributed; Sacrobosco died in 1256 and quotes the verse Algorism. Several commentaries on Alexander's verse treatise were composed, from one of which our first tractate was translated, and the text itself was from time to time enlarged, sections on proofs and on mental arithmetic being added. We have no indication of the source on which Alexander drew; it was most

likely one of the translations of Al-Khowarasmi, but he has also the Abacists in mind, as shewn by preserving the use of differences in multiplication. His treatise, first printed by Halliwell-Phillipps in his *Rara Mathematica*, is adapted for use on a board covered with sand, a method almost universal in the thirteenth century, as some passages in the algorism of that period already quoted show: "Est et alius modus qui utitur apud Indos, et doctor hujusmodi ipsos erat quidem nomine Algus. Et modus suus erat in computando per quasdam figuras scribendo in pulvere. . . ." "Si voluerimus depingere in pulvere predictos digitos secundum consuetudinem algorismi . . ." "et sciendum est quod in nullo loco minutorum sive secundorum . . . in pulvere debent scribi plusquam sexaginta."

Modern Arithmetic.

Modern Arithmetic begins with Leonardi Fibonacci's treatise "de Abaco," written in 1202 and re-written in 1228. It is modern rather in the range of its problems and the methods of attack than in mere methods of calculation, which are of its period. Its sole interest as regards the present work is that Leonardi makes use of the digital signs described in Record's treatise on *The arte of nombrynge by the hand* in mental arithmetic, calling it "modus Indorum." Leonardo also introduces the method of proof by "casting out the nines."

Digital Arithmetic.

The method of indicating numbers by means of the fingers is of considerable age. The British Museum possesses two ivory counters marked on one side by carelessly scratched Roman numerals IIIV and VIIII, and on the other by carefully engraved digital signs for 8 and 9. Sixteen seems to have been the number of a complete set. These counters were either used in games or for the counting board, and the Museum ones, coming from the Hamilton collection, are undoubtedly not later than the first century. Frohner has published in the *Zeitschrift des Münchener Alterthumsvereins* a set, almost complete, of them with a Byzantine treatise; a Latin treatise is printed among Bede's works. The use of this method is universal through

the East, and a variety of it is found among many of the native races in Africa. In medieval Europe it was almost restricted to Italy and the Mediterranean basin, and in the treatise already quoted (Sloane 3281) it is even called the Abacus, perhaps a memory of Fibonacci's work.

Methods of calculation by means of these signs undoubtedly have existed, but they were too involved and liable to error to be much used.

The Use of "Arabic" Figures.

It may now be regarded as proved by Bubnov that our present numerals are derived from Greek sources through the so-called Boethian "apices," which are first found in late tenth century manuscripts. That they were not derived directly from the Arabic seems certain from the different shapes of some of the numerals, especially the 0, which stands for 5 in Arabic. Another Greek form existed, which was introduced into Europe by John of Basingstoke in the thirteenth century, and is figured by Matthew Paris (V. 285); but this form had no success. The date of the introduction of the zero has been hotly debated, but it seems obvious that the twelfth century Latin translators from the Arabic were perfectly well acquainted with the system they met in their Arabic text, while the earliest astronomical tables of the thirteenth century I have seen use numbers of European and not Arabic origin. The fact that Latin writers had a convenient way of writing hundreds and thousands without any cyphers probably delayed the general use of the Arabic notation. Dr. Hill has published a very complete survey of the various forms of numerals in Europe. They began to be common at the middle of the thirteenth century and a very interesting set of family notes concerning births in a British Museum manuscript, Harl. 4350 shows their extension. The first is dated Mij^{c}. lviii., the second Mij^{c}. lxi., the third Mij^{c}. 63, the fourth 1264, and the fifth 1266. Another example is given in a set of astronomical tables for 1269 in a manuscript of Roger Bacon's works, where the scribe began to write MCC6. and crossed out the figures, substituting the "Arabic" form.

The Counting Board.

The treatise on pp. 52-65 is the only one in English known on the subject. It describes a method of calculation which, with slight modifications, is current in Russia, China, and Japan, to-day, though it went out of use in Western Europe by the seventeenth century. In Germany the method is called "Algorithmus Linealis," and there are several editions of a tract under this name (with a diagram of the counting board), printed at Leipsic at the end of the fifteenth century and the beginning of the sixteenth. They give the nine rules, but "Capitulum de radicum extractione ad algoritmum integrorum reservato, cujus species per ciffrales figuras ostenduntur ubi ad plenum de hac tractabitur." The invention of the art is there attributed to Appulegius the philosopher.

The advantage of the counting board, whether permanent or constructed by chalking parallel lines on a table, as shown in some sixteenth-century woodcuts, is that only five counters are needed to indicate the number nine, counters on the lines representing units, and those in the spaces above representing five times those on the line below. The Russian abacus, the "tchatui" or "stchota" has ten beads on the line; the Chinese and Japanese "Swanpan" economises by dividing the line into two parts, the beads on one side representing five times the value of those on the other. The "Swanpan" has usually many more lines than the "stchota," allowing for more extended calculations, see Tylor, *Anthropology* (1892), p. 314.

Record's treatise also mentions another method of counter notation (p. 64) "merchants' casting" and "auditors' casting." These were adapted for the usual English method of reckoning numbers up to 200 by scores. This method seems to have been used in the Exchequer. A counting board for merchants' use is printed by Halliwell in *Rara Mathematica* (p. 72) from Sloane MS. 213, and two others are figured in Egerton 2622 f. 82 and f. 83. The latter is said to be "novus modus computandi secundum inventionem Magistri Thome Thorleby," and is in principle, the same as the "Swanpan."

The Exchequer table is described in the *Dialogus de Scaccario* (Oxford, 1902), p. 38.

1. Halliwell printed the two sides of his leaf in the wrong order. This and some obvious errors of transcription—'ferye' for 'ferthe,' 'lest' for

‘left,’ etc., have not been corrected in the reprint on pp. 70-71.

2. For Egyptian use see Herodotus, ii. 36, Plato, *de Legibus*, VII.

3. See on this Dr. Poole, *The Exchequer in the Twelfth Century*, Chap. III., and Haskins, *Eng. Hist. Review*, 27, 101. The hidage of Essex in 1130 was 2364 hides.

4. These figures are removed at the next step.

5. Slates are mentioned by Chaucer, and soon after (1410) Prosdocimo de Beldamandi speaks of the use of a “lapis” for making notes on by calculators.

The Earliest Arithmetics in English.

The Crafte of Nombrynge.

Egerton 2622.

leaf 136 *a*.

HEc algorism*us* ars p*re*sens dicit*ur*; in qua
Talib*us* indor*um* fruim*ur* bis qui*n*q*ue* figuris.

A derivation of Algorism. This boke is called þe boke of algorym, or Augrym aft*er* lewd*er* vse. And þis boke tretys þe Craft of Nombryng, þe quych crafte is called also Algorym. Ther was a kyng of Inde, þe quich heyth Algor, & he made þis craft. And aft*er* his name he called hit algory*m*; Another derivation of the word. or els another cause is quy it is called Algorym, for þe latyn word of hit s. Algorism*us* come*s* of Algos, grece, q*uid* e*st* ars, latine, craft oɳ englis, and rides, q*uid* e*st* nu*me*r*us*, latine, A nomb*ur* oɳ englys, inde d*icitu*r Algorism*us* p*er* addic*i*one*m* hui*us* sillabe m*us* & subtracc*i*onem d & e, q*ua*si ars num*er*andi.
¶ fforthermor*e* ȝe most vnd*ir*stonde þ*a*t in þis craft ben vsid teen figurys, as here ben*e* writen for ensampul, φ 9 8 7 6 5 4 3 2 1. ¶ Expone þe too v*er*sus afor*e*: this p*re*sent craft ys called Algorism*us*, in þe quych we vse teen signys of Inde. Questio. ¶ Why teɳ fyguris of Inde? Solucio. for as I haue sayd afore þai were fonde fyrst in Inde of a kyng*e* of þat Cuntre, þ*a*t was called Algor.

ion and Numeration.

v*ersus* [in margin].

¶ Prima sig*nifica*t unu*m*; duo ve*r*o s*ec*un*d*a:
¶ Tercia sig*nifica*t tria; sic procede sinistre.

¶ Don*e* ad extrema*m* venias, que cifra voca*tur*.

¶ Cap*itulu*m primum de significac*ion*e figurar*um*.

Expos*itio versus*. In þis verse is notifide þe significac*i*on of þese figur*is*. And þus expone the verse. The meaning and place of the figures. Þe first signifiyth on*e*, þe secu*n*de leaf 136 *b.* signi*fiyth tweyn*e, þe thryd signifiyth thre, & the fourte signifiyth 4. ¶ And so forthe towarde þe lyft syde of þe tabul or of þe boke þ*a*t þe figures ben*e* writen*e* in, til þat þ*o*u come to the last figure, þ*a*t is called a cifre. ¶ Questio. In quych syde sittes þe first figur*e*? Soluc*io*, forsothe loke quich figure is first in þe ry3t side of þe bok or of þe tabul, & þ*a*t same is þe first figur*e*, for þ*o*u schal write bakeward, as here, 3. 2. 6. 4. 1. 2. 5. Which figure is first. The figure of 5. was first write, & he is þe first, for he sittes on̨ þe ri3t syde. And the fig*ur*e of 3 is last. ¶ Neu*er*-þe-les wen he says ¶ P*ri*ma sig*nific*at vnu*m* &c., þat is to say, þe first betokenes on*e*, þe secu*n*de. 2. & fore-þ*er*-mor*e*, he vnd*ir*stondes no3t of þe first fig*ur*e of eu*er*y rew. ¶ But he vnd*ir*stondes þe first figure þ*a*t is in þe nomb*ur* of þe forsayd teen figuris, þe quych is on*e* of þ*e*se. 1. And þe secu*n*de 2. & so forth.

v*ersus* [in margin].

¶ Quelib*et* illar*um* si pr*im*o limite ponas,
¶ Simpliciter se significat: si v*er*o sec*un*do,
Se decies: sursu*m* *pr*ocedas m*u*ltiplicando.
¶ Na*m*q*ue* figura sequ*e*ns q*uam*uis signat decies pl*us*.
¶ Ipsa locata loco quam sign*ific*at p*ertin*ente.

Expos*itio* [in margin]. ¶ Expone his v*er*se þus. Eu*er*y of þese figuris bitokens hym selfe & no mor*e*, yf he stonde in þe first place of þe rewele / this worde Simplicit*er* in þat verse it is no more to say but þat, & no mor*e*. An explanation of the principles of notation in ¶ If it stonde in the secu*n*de place of þe rewle, he betokens ten*e* tymes hym selfe, as þis figur*e* 2 here 20 tokens ten tyme hym selfe, leaf 137 *a.* *þat is twenty, for he hym selfe betokenes twey*ne*, & ten tymes twene is twenty. And for he stondis on̨ þe lyft side

& in þe secu*n*de place, he betokens ten tyme hy*m* selfe. And so go forth. ¶ ffor eu*er*y fig*ure*, & he stonde aft*ur* a-noþ*er* toward the lyft side, he schal betoken*e* ten tymes as mich mor*e* as he schul betoken & he stode in þe place þ*ere* þat þe fig*ure* a-for*e* hym stondes. An exampulle. 9. 6. 3. 4. Þe fig*ure* of 4. þ*a*t hase þis schape {4}. betokens bot hymselfe, for he stondes in þe first place. The fig*ure* of 3. þat hase þis schape {3}. betokens ten tymes mor*e* þen he schuld & he stde þ*ere* þ*a*t þe fig*ure* of 4. stondes, þ*a*t is thretty. The fig*ure* of 6, þ*a*t hase þis schape {6}, betokens ten tymes mor*e* þan he schuld & he stode þ*ere* as þe fig*ure* of {3}. stondes, for þ*ere* he schuld tokyn*e* bot sexty, & now he betokens ten tymes mor*e*, þat is sex hundryth. The fig*ure* of 9. þ*a*t hase þis schape {9}. betokens ten tymes mor*e* þan*e* he schuld & he stode in þe place þ*ere* þe fig*ure* of sex stondes, for þen he schuld betoken to 9. hundryth, and in þe place þ*ere* he stondes now he betokens 9. þousande. And þe hole nomb*ur* is 9 thousande sex hundryth & four*e* & thretty. ¶ ffor thermor*e*, when þ*o*u schalt rede a nomb*ur* of fig*ure*, þ*o*u schalt begyn*e* at þe last fig*ure* in the lyft side, & rede so forth to þe riȝt side as her*e* 9. 6. 3. 4. Thou schal begyn to rede at þe fig*ure* of 9. & rede forth þus. 9. leaf 137 *b.* *thousand sex hundryth thritty & foure. But when þ*o*u schall*e* write, þ*o*u schalt be-gynne to write at þe ryȝt side.

¶ Nil cifra sig*nifica*t s*ed* dat signa*re* sequenti.

The meaning and vse of the cipher. Expone þis vers*e*. A cifre tokens noȝt, bot he makes þe fig*ure* to betoken þat comes aft*ur* hym mor*e* þan he schuld & he were away, as þus 1φ. her*e* þe fig*ure* of on*e* tokens ten, & yf þe cifre were away [1] & no fig*ure* by-for*e* hym he schuld token bot on*e*, for þan he sch*ul*d stonde in þe first place. ¶ And þe cifre tokens nothyng hym selfe. for al þe nomb*ur* of þe ylke too fig*ure*s is bot ten. ¶ Questio. Why says he þat a cifre makys a fig*ure* to signifye (tyf) mor*e* &c. ¶ I speke for þis worde significatyf, ffor sothe it may happe aft*ur* a cifre schuld come a-noþ*ur* cifre, as þus 2φφ. And ȝet

þe secunde cifre shuld token neu*er* þe mor*e* excep he schuld kepe þe ord*er* of þe place. and a cifre is no fig*ure* significatyf.

¶ Q*ua*m p*re*cedentes plus ulti*m*a significabit /

The last figure means more than all the others, since it is of the highest value.

Expone þis v*er*se þus. Þe last figu*re* schal token mor*e* þan all*e* þe oþ*er* afor*e*, thouȝt þ*ere* wer*e* a hundryth thousant figures afor*e*, as þus, 16798. Þe last fig*ure* þat is 1. betokens ten thousant. And all*e* þe oþ*er* figu*re*s b*e*n bot betoken*e* bot sex thousant seuyn*e* h*u*ndryth nynty & 8. ¶ And ten thousant is mor*e* þen all*e* þat nomb*ur*, *er*go þe last figu*re* tokens mor*e* þan all þe nomb*ur* afor*e*.

hree Kinds of Numbers

leaf 138 *a*.

*** ¶ Post p*re*dicta scias breuit*er* q*uod* tres num*er*or*um* Distincte species sunt; nam quidam digiti sunt; Articuli quidam; quidam q*uoque* compositi sunt.**

¶ Capit*ul*um 2^m de t*ri*plice divisione nu*mer*or*um*.

¶ The auctor of þis tretis dep*ar*tys þis worde a nomb*ur* into 3 p*ar*tes. Some nomb*ur* is called digit*us* latine, a digit in englys. Some nomb*ur* is called articul*us* latine. An Articul in englys. Some nomb*ur* is called a composyt in englys. ¶ Expone þis v*er*se. know þ*o*u aft*ur* þe forsayd rewles þ*a*t I sayd afore, þat þ*ere* ben thre spices of nomb*ur*. Oon*e* is a digit, Anoþ*er* is an Articul, & þe toþ*er* a Composyt. v*er*sus.

Digits.

Articles.

Composites.

, Articles, and Composites.

¶ Sunt digiti num*er*i qui cit*ra* denariu*m* s*u*nt.

What are digits.

¶ Her*e* he telles qwat is a digit, Expone v*er*su*s* sic. Nomb*ur*s digitus ben*e* all*e* nomb*ur*s þat ben w*ith*-inne ten, as nyne, 8. 7. 6. 5. 4. 3. 2. 1.

¶ Articupli decupli degito*rum*; compositi s*u*nt
Illi qui constant ex articulis degitisq*ue*.

¶ Her*e* he telles what is a composyt and what is an*e* articul. Expone sic v*er*sus. ¶ Articulis ben[2] all*e* þ*a*t may be deuidyt into nomb*urs* of ten & nothyng*e* leue ou*er*, as twenty, thretty, fourty, a hundryth, a thousand, & such oþ*er*, ffor twenty may be dep*ar*tyt in-to 2 nomb*ur*s of ten, fforty in to four*e* nomb*ur*s of ten, & so forth.

What are articles.

leaf 138 *b*.

What numbers are composites.

*Compositys ben nomb*ur*s þat bene componyt of a digyt & of an articull*e* as fouretene, fyftene, sextene, & such oþ*er*. ffortene is co*m*ponyd of four*e* þat is a digit & of ten þat is an articull*e*. ffiftene is componyd of 5 & ten, & so of all oþ*er*, what þat þai ben. Short-lych eu*er*y nomb*ur* þat be-gynnes w*ith* a digit & endyth in a articull*e* is a composyt, as fortene bygennyng*e* by four*e* þat is a digit, & endes in ten.

¶ Ergo, p*ro*posito nu*mer*o tibi scriber*e*, p*ri*mo
Respicias quid sit nu*merus*; si digitus sit
P*ri*mo scribe loco digitu*m*, si compositus sit
P*ri*mo scribe loco digitu*m* post articulu*m*; sic.

How to write a number.

¶ here he telles how þ*o*u schalt wyrch whan þ*o*u schalt write a nomb*ur*. Expone v*er*su*m* sic, & fac iuxta expon*ent*is sentencia*m*; whan þ*o*u hast a nomb*ur* to write, loke fyrst what man*er* nomb*ur* it ys þ*a*t þ*o*u schalt write, whether it be a digit or a composit or an Articul. ¶ If he be a digit, write a digit, as yf it be seuen, write seuen & write þ*a*t digit in þe first place toward þe ryght side. If it be a composyt, write þe digit of þe composit in þe first place & write þe articul of þat digit in þe secunde place next toward þe lyft side. As yf þ*o*u schal write sex & twenty. write þe digit of þe nomb*ur* in þe first place þat is sex, and write þe articul next aft*ur* þat is twenty, as þus 26. But whan þou schalt sowne or speke *or rede an Composyt þou schalt first sowne þe articul &

If it is a digit.

If it is a composite.

How to read it.

leaf 139 *a*.

aft*ur* þe digit, as þ*o*u seyst by þe comyn*e* speche, Sex & twenty & nouȝt twenty & sex. v*er*sus.

¶ Articul*us* si sit, in p*ri*mo limite cifram,
Articulu*m vero* reliq*ui*s insc*ri*be figur*is*.

How to write Articles.

¶ Here he tells how þ*o*u schal write when þe nombre þ*a*t þ*o*u hase to write is an Articul. Expone v*er*sus sic & fac s*ecundu*m sentenciam. Ife þe nomb*ur* þ*a*t þ*o*u hast write be an Articul, write first a cifre & aft*ur* þe cifer write an Articul*le* thus. 2φ. ffurthermore, þ*o*u schalt vnd*ir*stonde yf þ*o*u haue an Articul, loke how mych he is, yf he be w*ith*-ynne an hundryth, þ*o*u schalt write bot on*e* cifre, afore, as her*e* .9φ.

Tens,

If þe articulle be by hym-silfe & be an hundrid euen*e*, þen schal þ*o*u write .1. & 2 cifers afor*e*, þat he may stonde in þe thryd place, for eu*er*y fig*ure* in þe thryd place schal token a hundrid tymes hym selfe. If þe articul be a thousant or thousandes[3] and he stonde by hy*m* selfe, write afor*e* 3 cifers & so forþ of al oþ*er*.

hundreds,

thousands, &c.

¶ Quolib*et* in nu*mer*o, si par sit p*ri*ma figura,
Par erit & to*tu*m, quicquid sibi co*nti*n*ua*t*ur*;
Imp*ar* si fu*er*it, totu*m* tu*n*c fiet *et* impar.

To tell an even number

¶ Here he teches a gen*er*all*e* rewle þ*a*t yf þe first fig*ure* in þe rewle of fig*ure*s token a nomb*ur* þat is euen*e* al þ*a*t nomb*ur* of fig*ur*ys in þat rewle schal be euen*e*, as her*e* þ*o*u may see 6. 7. 3. 5. 4. Computa & p*ro*ba. ¶ If þe first *fig*ur*e token an nomb*ur* þat is ode, all*e* þat nomb*ur* in þat rewle schall*e* be ode, as her*e* 5 6 7 8 6 7. Computa & p*ro*ba. v*er*sus.

Or an odd.

*leaf 139 *b*.

¶ Septe*m* su*n*t partes, no*n* pl*u*res, istius artis;
¶ Adder*e*, subt*ra*her*e*, duplar*e*, dimidiar*e*,
Sextaq*ue* diuider*e*, s*ed* qui*n*ta m*u*ltiplicar*e*;
Radice*m* ext*ra*her*e* p*ar*s septi*m*a dicitur esse.

even Rules of Arithmetic.

The seven rules. ¶ Here telles þ*a*t þ*er* ben .7. spices or p*ar*tes of þis craft. The first is called addicioñ, þe secunde is called subtraccioñ. The thryd is called duplacioñ. The 4. is called dimydicioñ. The 5. is called m*u*ltiplicacioñ. The 6 is called diuisioñ. The 7. is called extraccioñ of þe Rote. What all þese spices ben*e* hit schall*e* be tolde singillati*m* in her*e* caputul*e*.

¶ Subt*ra*his aut addis a dext*ri*s vel mediabis:

Add, subtract, or halve, from right to left. Thou schal be-gynne in þe ryght side of þe boke or of a tabul. loke wer*e* þ*o*u wul be-gynne to write latyn or englys in a boke, & þ*a*t schall*e* be called þe lyft side of the boke, þat þ*o*u writest toward þ*a*t side schal be called þe ryght side of þe boke. V*er*sus.

A leua dupla, diuide, m*u*ltiplica.

Here he telles þe in quych side of þe boke or of þe tabul þ*o*u schall*e* be-gyn*e* to wyrch duplacioñ, diuisioñ, and m*u*ltiplicacioñ. Multiply or divide from left to right. Thou schal begyn*e* to worch in þe lyft side of þe boke or of þe tabul, but yn what wyse þ*o*u schal wyrch in hym **dicetur singillatim in seque*n*tib*us* capi*tulis* et de vtilitate cui*us*li*bet* art*is* & sic Completur** leaf 140. ***p*ro*hemi*um* & sequit*ur* tractat*us* & p*ri*mo de arte addic*ion*is que p*ri*ma ars est in ordine.**

Craft of Addition.

Adder*e* si nu*mer*o num*e*ru*m* vis, ordine tali
Incipe; scribe duas p*rim*o series nu*mer*or*um*
P*ri*ma*m* sub p*ri*ma recte pone*n*do figura*m*,
Et sic de reliq*ui*s facias, si sint tibi plures.

Four things must be known: ¶ Her*e* by-gynnes þe craft of Addicioñ. In þis craft þ*o*u most knowe foure thyng*es*. ¶ Fyrst þ*ou* most know what is addicioñ. Next þ*o*u most know how mony rewles of figurys þou most haue. ¶ Next þ*o*u most know how mony diue*r*s casys happes in þis craft of addicioñ. ¶ And next qwat

is þe p*ro*fet of þis craft. ¶ As for þe first þou most know þat addicioñ is a castyng to-ged*ur* of twoo nomburys in-to on*e* nombr*e*. As yf I aske qwat is twene & thre. Þ*o*u wyl cast þese twene nomb*re*s to-ged*ur* & say þ*a*t it is fyue. how many ¶ As for þe secunde þou most know þ*a*t þou schall*e* haue tweyne rewes of figures, on*e* vndur a-nother, as her*e* þ*o*u mayst se. 1234

2168. ¶ As for þe thryd þou most know þ*a*t ther*e* ben foure diu*er*se cases. As for þe forthe þ*o*u most know þ*a*t þe p*ro*fet of þis craft is to telle what is þe hole nomb*ur* þ*a*t comes of diu*er*se nomburis. Now as to þe texte of oure verse, he teches ther*e* how þ*o*u schal worch in þis craft. ¶ He says yf þ*o*u wilt cast on*e* nomb*ur* to anoþ*er* nomb*ur*, þou most by-gynne on þis wyse. ¶ ffyrst write *two rewes of figuris & nombris so þat þ*o*u write þe first figur*e* of þe hyer nomb*ur* euen*e* vnd*ir* the first fig*ure* of þe nether nomb*ur*, 123

234. And þe secunde of þe nether nomb*ur* euen*e* vnd*ir* þe secunde of þe hyer, & so forthe of eu*er*y fig*ur*e of both þe rewes as þ*o*u mayst se.

Cases of the Craft of Addition.

¶ Inde duas adde p*ri*mas hac condic*i*one:
Si digitus crescat ex addic*i*one prior*um*;
P*ri*mo scribe loco digitu*m*, quicu*n*q*ue* sit ille.

¶ Here he teches what þ*o*u schalt do when þ*o*u hast write too rewes of figuris on vnder an-oþ*er*, as I sayd be-fore. ¶ He says þou schalt take þe first fig*ur*e of þe heyer nomb*re* & þe fyrst figur*e* of þe neþ*er* nombre, & cast hem to-ged*er* vp-on þis condicion. Thou schal loke qweþ*er* þe nomber þat comys þ*ere*-of be a digit or no. ¶ If he be a digit þ*o*u schalt do away þe first fig*ur*e of þe hyer nomb*re*, and write þ*ere* in his stede þat he stode Inne þe digit, þ*a*t comes of þe ylke 2 fig*ur*es, & so wrich forth on oþ*er* figures yf þ*ere* be ony moo, til þ*o*u come to þe ende toward þe lyft side. And

lede þe nether fig*ure* stonde still eu*er*-mor*e* til þ*o*u haue ydo. ffor þ*ere*-by þ*o*u schal wyte whe*þer* þ*o*u hast don*e* wel or no, as I schal tell þe aft*er*ward in þe ende of þis Chapt*er*. ¶ And loke allgate leaf 141 *a.* þat þou be-gynne to worch in þis Craft of Addi*cion in þe ryȝt side, here is an exampul of þis case. 1234

2142. Caste 2 to four*e* & þat wel be sex, do away 4. & write in þe same place þe fig*ur*e of sex. ¶ And lete þe fig*ur*e of 2 in þe nether rewe stonde stil. When þ*o*u hast do so, cast 3 & 4 to-ged*ur* and þat wel be seuen þ*a*t is a digit. Do away þe 3, & set þ*ere* seuen, and lete þe neþ*er* fig*ure* stonde still*e*, & so worch forth bakward til þ*o*u hast ydo all to-ged*er*.

Et si composit*us*, in limite scribe sequ*en*te
Articulum, p*ri*mo digitum; q*uia* sic iubet ordo.

¶ Here is þe secunde case þ*a*t may happe in þis craft. And þe case is þis, Suppose it is a Composite, set down the digit, and carry the
f þe casting of 2 nomburis to-ged*er*, as of þe fig*ur*e of þe hyer rewe & of þe figure of þe neþ*er* rewe come a Composyt, how schalt þ*ou* worch. Þ*us* þ*o*u schalt worch. Thou shalt do away þe fig*ur*e of þe hyer nomb*er* þat was cast to þe figure of þe neþ*er* nomber. ¶ And write þ*ere* þe digit of þe Composyt. And set þe articul of þe composit next aft*er* þe digit in þe same rewe, yf þ*ere* be no mo fig*ur*es aft*er*. But yf þ*ere* be mo figuris aft*er* þat digit. And þere he schall be rekend for hym selfe. And when þ*o*u schalt adde þ*a*t ylke figure þ*a*t berys þe articull*e* ou*er* his hed to þe figur*e* vnd*er* hym, þ*o*u schalt cast þat articul to þe figure þ*a*t hase hym ou*er* his hed, & þ*ere* þat Articul schal token hym selfe. here is an Ensampull leaf 141 *b.* *of all. 326

216. Cast 6 to 6, & þ*ere*-of wil arise twelue. do away þe hyer 6 & write þ*ere* 2, þ*a*t is þe digit of þis composit. And þe*n* write þe articull*e* þat is ten ou*er* þe figuris hed of twene as þ*us*. 1

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216. Now cast þe articull*e* þ*a*t standus vpon þe fig*ur*is of

twene hed to þe same fig*ur*e, & reken þat articul bot for on*e*, and þan þ*ere* wil arise thre. Þan cast þat thre to þe neþ*er* figure, þat is on*e*, & þat wul be four*e*. do away þe fig*ur*e of 3, and write þ*ere* a fig*ur*e of foure. and lete þe neþ*er* fig*ur*e stonde stil, & þan worch forth. vn*de ver*sus.

¶ Articulus si sit, in p*ri*mo limite cifram,
¶ Articulu*m* v*er*o reliquis inscribe figuris,
Vel p*er* se scribas si nulla figura sequat*ur*.

¶ Her*e* he puttes þe thryde case of þe craft of Addicioŋ. & þe case is þis. Suppose it is an Article, set down a cipher and carry the tens yf of Addiciouŋ of 2 figuris a-ryse an Articull*e*, how schal þ*o*u do. thou most do away þe heer fig*ur*e þ*a*t was addid to þe neþ*er*, & write þ*ere* a cifre, and sett þe articuls on þe figuris hede, yf þ*a*t þ*ere* come ony aft*er*. And wyrch þan as I haue tolde þe in þe secunde case. An ensampull. 25.
15 Cast 5 to 5, þat wylle be ten. now do away þe hyer 5, & write þ*ere* a cifer. And sette ten vpon þe figuris hed of 2. And reken it but for on þus. lo Here is an example an Ensampull*e* 1
2φ
15 And leaf 142 *a*. *þan worch forth. But yf þ*ere* come no figure aft*er* þe cifre, write þe articul next hym in þe same rewe as here 5
5 cast 5 to 5, and it wel be ten. do away 5. þat is þe hier 5. and write þ*ere* a cifre, & write aft*er* hym þe articul as þus
1φ
5 And þan þ*o*u hast done.

¶ Si tibi cifra sup*er*ueniens occurrerit, illa*m*
Dele sup*er*posita*m*; fac illic scribe figura*m*,
Postea procedas reliquas addendo figuras.

What to do when you have a cipher in the top row ¶ Here he putt*es* þe fourt case, & it is þis, þat yf þ*ere* come a cifer in þe hier rewe, how þ*o*u schal do. þus þ*o*u schalt do. do away þe cifer, & sett þ*ere* þe digit þ*a*t comes of þe addiciou*n* as þus 1φφ84.
17743 An example of all the difficulties. In þis ensampul ben all*e* þe four*e* cases.

Cast 3 to foure, þ*a*t wol be seuen. do away 4. & write þ*ere* seuen; þan cast 4 to þe figur*e* of 8. þ*a*t wel be 12. do away 8, & sett þ*ere* 2. þat is a digit, and sette þe articul of þe composit, þat is ten, vpon þe cifers hed, & reken it for hym selfe þat is on. þan cast on*e* to a cifer, & hit wull*e* be but on, for no3t & on makes but on*e*. þan cast 7. þ*a*t stondes vnd*er* þat on to hym, & þat wel be 8. do away þe cifer & þat 1. & sette þ*ere* 8. þan go forthermor*e*. cast þe oþ*er* 7 to þe cifer þ*a*t stondes ou*er* hy*m*. þ*a*t wul be bot seuen, for þe cifer betokens no3t. do away þe cifer & sette þ*ere* seuen, leaf 142 *b*. *& þen go forþ*er*mor*e* & cast 1 to 1, & þat wel be 2. do away þe hier 1, & sette þ*ere* 2. þan hast þ*o*u do. And yf þ*o*u haue wel ydo þis nomber þat is sett her*e*-aft*er* wel be þe nomber þat schall*e* aryse of all*e* þe addicion. as her*e* 27827. ¶ Sequi*tu*r alia sp*eci*es.

raft of Subtraction.

A nu*mer*o num*er*u*m* si sit tibi demer*e* cura
Scribe figurar*um* series, vt in addicione.

Four things to know about subtraction.

¶ This is þe Chapt*er* of subtraccion, in the quych þou most know foure nessessary thyng*es*. the first what is subtraccion. þe secunde is how mony nombers þou most haue to subt*ra*ccion, the thryd is how mony maners of cases þ*ere* may happe in þis craft of subtraccion. The fourte is qwat is þe p*ro*fet of þis craft. ¶ As for þe first, þou most know þ*a*t subtraccion is drawyng*e* of on*e* nowmb*er* oute of anoþ*er* nombr.

As for the first, þou most know

As for þe secunde, þou most knowe þ*a*t þou most haue two rewes of figuris on*e* vnd*er* anoþ*er*, as þ*o*u addyst in addicion.

As for þe secunde,

As for þe thryd, þ*o*u moyst know þ*a*t four*e* man*er* of diu*er*se casis mai happe in þis craft.

As for þe thryd,

¶ As for þe fourt, þou most know þ*a*t þe p*ro*fet of þis craft is whenne þ*o*u hasse taken þe lasse nomber out of þe mor*e* to telle what þ*ere* leues ou*er* þ*a*t. & þ*o*u most be-gynne to wyrch in þ*is* craft in þe ryght side of þe boke, as þ*o*u diddyst in addicion. V*er*sus.

As for þe fourt,

¶ Maiori nu*mer*o num*eru*m suppone minorem,
¶ Siue pari nu*mer*o supponat*ur* num*er*us par.

leaf 143 *a*. Put the greater number above the less.

* ¶ Her*e* he telles þat þe hier nomber most be mor*e* þen þe neþ*er*, or els euen̨ as mych. but he may not be lasse. And þe case is þis, þou schalt drawe þe neþ*er* nomber out of þe hyer, & þou mayst not do þ*a*t yf þe hier nomber wer*e* lasse þan þat. ffor þ*o*u mayst not draw sex out of 2. But þ*o*u mast draw 2 out of sex. And þou maiste draw twene out of twene, for þou schal leue noȝt of þe hier twene vn*de* ver*s*us.

Cases of the Craft of Subtraction.

¶ Postea si possis a prima subt*ra*he p*ri*ma*m*
Scribens quod remanet.

The first case of subtraction.

Here is þe first case put of subtraccion̨, & he says þou schalt begynne in þe ryght side, & draw þe first fig*ure* of þe neþ*er* rewe out of þe first fig*ure* of þe hier rewe. qwether þe hier fig*ur*e be mor*e* þen þe neþ*er*, or euen̨ as mych. And þat is notified in þe vers when he says “Si possis.” Whan þ*o*u has þus ydo, do away þe hiest fig*ur*e & sett þ*ere* þat leues of þe subtraccion̨, lo an Ensampull*e*

Here is an example.

234
122

draw 2 out of 4. þan leues 2. do away 4 & write þ*ere* 2, & latte þe neþ*er* figur*e* sto*n*de stille, & so go for-by oþ*er* figuris till þ*o*u come to þe ende, þan hast þ*o*u do.

¶ Cifram si nil remanebit.

Put a cipher if nothing remains.

¶ Here he putt*es* þe secunde case, & hit is þis. yf it happe þ*a*t qwen þ*o*u hast draw on neþ*er* fig*ure* out of a hier, & þ*er*e leue noȝt aft*er* þe subt*ra*ccion̨, þus leaf 143 *b*. *þou schalt do. þ*o*u schall*e* do away þe hier fig*ur*e & write þ*ere* a cifer, as lo an Ensampull

Here is an example.

24
24

Take four*e* out of four*e* þan leus noȝt. þ*er*efor*e* do away þe hier 4 & set þ*ere* a cifer, þan take 2 out of 2, þan leues

no3t. do away þe hier 2, & set þ*ere* a cifer, and so worch whar*e* so eu*er* þis happe.

Sed si no*n* possis a p*ri*ma dem*er*e p*ri*ma*m*
P*re*cedens vnu*m* de limite deme sequ*en*te,
Quod demptu*m* p*ro* denario reputabis ab illo
Subt*ra*he tot*a*lem num*er*u*m* qu*em* p*ro*posuisti
Quo facto sc*ri*be super quicquid remaneb*it*.

Suppose you cannot take the lower figure from the upper, borrow. Here he puttes þe thryd case, þe quych is þis. yf it happe þat þe neþ*er* fig*ur*e be mor*e* þen þe hier fig*ur*e þat he schall*e* be draw out of. how schall*e* þou do. þus þ*o*u schall*e* do. þou schall*e* borro .1. oute of þe next fig*ur*e þat comes aft*er* in þe same rewe, for þis case may neu*er* happ but yf þ*ere* come figures aft*er*. þan þ*o*u schalt sett þat on ou*er* þe hier figur*es* hed, of the quych þou woldist y-draw oute þe neyþ*er* fig*ur*e yf þ*o*u haddyst y-my3t. Whane þou hase þus ydo þou schall*e* rekene þ*a*t .1. for ten.

take the lower number from ten ¶ And out of þat ten þ*o*u schal draw þe neyþermost fig*ur*e, And all*e* þ*a*t leues þou schall*e* add the answer to the upper figure adde to þe figur*e* on whos hed þat .1. stode. And þen þ*o*u schall*e* do away all*e* þat, & sett þ*ere* all*e* that arisys of the addicion of þe ylke 2 fig*ur*is. And yf yt leaf 144 *a.* *happe þat þe fig*ur*e of þe quych þ*o*u schalt borro on be hym self but 1. If þ*o*u schalt þat on*e* & sett it vppon þe oþ*er* figur*is* hed, and sett in þ*a*t 1. place a cifer, yf þ*ere* come mony figur*es* after. Exampul. 2122 1134 take 4 out of 2. it wyl not be, þerfor*e* borro on*e* of þe next figur*e*, þ*a*t is 2. and sett þat ou*er* þe hed of þe fyrst 2. & rekene it for ten. and þere þe secunde stondes write 1. for þ*o*u tokest on out of hy*m*. þan take þe neþ*er* fig*ur*e, þat is 4, out of ten. And þen leues 6. cast to 6 þe fig*ur*e of þat 2 þat stode vnd*er* þe hedde of 1. þat was borwed & rekened for ten, and þat wylle be 8. do away þ*a*t 6 & þat 2, & sette þ*ere* 8, & lette þe neþ*er* fig*ur*e stonde stille. Whanne þ*o*u hast do þus, go to þe next fig*ur*e þ*a*t is now bot 1. but first yt was 2, & þ*ere*-of was borred 1. How to ‘Pay back’ the borrowed ten þan take out of þ*a*t

þe fig*ur*e vnd*er* hym, þ*a*t is 3. hit wel not be. þer-fo*re* borowe of the next fig*ur*e, þe quych is bot 1. Also take & sett hym ou*er* þe hede of þe fig*ure* þat þou woldest haue y-draw oute of þe nether figure, þe quych was 3. & þou myȝt not, & rekene þ*a*t borwed 1 for ten & sett in þe same place, of þe quych place þ*o*u tokest hy*m* of, a cifer, for he was bot 1. Whanne þ*o*u hast þ*us* ydo, take out of þat 1. þ*a*t is rekent for ten, þe neþ*er* figure of 3. And þ*ere* leues 7. leaf 144 *b.* *cast þe ylke 7 to þe fig*ur*e þat had þe ylke ten vpon his hed, þe quych fig*ur*e was 1, & þat wol be 8. þan do away þ*a*t 1 and þ*a*t 7, & write þ*ere* 8. & þan wyrch forth in oþ*er* figuris til þ*o*u come to þe ende, & þan þ*o*u hast þe do. V*er*sus.

¶ Facque nonenarios de cifris, cu*m* remeabis
¶ Occ*ur*rant si forte cifre; dum demps*er*is vnum
¶ Postea p*ro*cedas reliquas deme*n*do figuras.

A very hard case is put*es* ¶ Here he putt*es* þe fourte case, þe quych is þis, yf it happe þat þe neþ*er* fig*ur*e, þe quych þ*o*u schalt draw out of þe hier fig*ur*e be mor*e* pan þe hier figur ou*er* hym, & þe next fig*ur*e of two or of thre or of foure, or how mony þ*ere* be by cifers, how wold þ*o*u do. Þ*o*u wost wel þ*o*u most nede borow, & þ*o*u mayst not borow of þe cifers, for þai haue noȝt þat þai may lene or spar*e*. Ergo [4] how woldest þ*o*u do. Certayn̨ þus most þ*o*u do, þ*o*u most borow on of þe next figure significatyf in þat rewe, for þis case may not happe, but yf þ*ere* come figures significatyf aft*er* the cifers. Whan þ*o*u hast borowede þ*a*t 1 of the next figure significatyf, sett þ*a*t on ou*er* þe hede of þ*a*t fig*ur*e of þe quych þ*o*u wold haue draw þe neþ*er* figure out yf þ*o*u hadest myȝt, & reken it for ten as þ*o*u diddest i*n* þe oþ*er* case her*e*-a-for*e*. Whan̨ þ*o*u hast þus y-do loke how mony cifers þ*ere* were bye-twene þat figur*e* significatyf, & þe fig*ur*e of þe quych þ*o*u woldest haue y-draw the leaf 145 *a.* *neþ*er* figure, and of eu*er*y of þe ylke cifers make a figur*e* of 9. Here is an Ensample.

40002
10004 Take 4 out of 2. it wel not be. borow 1 out of be next

figure significatyf, þe quych is 4, & þen leues 3. do away þ*a*t figur*e* of 4 & write þ*ere* 3. & sett þ*a*t 1 vppon þe fig*ur*e of 2 hede, & þan take 4 out of ten, & þan þere leues 6. Cast 6 to the fig*ur*e of 2, þ*a*t wol be 8. do away þat 6 & write þ*er*e 8. Whan þ*o*u hast þus y-do make of eu*er*y 0 betweyn 3 & 8 a figure of 9, & þan worch forth in godes name. Sic. 10004 & yf þ*o*u hast wel y-do þ*o*u [5] schalt haue þis nomb*er*

to prove the Subtraction.

¶ Si subt*ra*cc*io* sit b*ene* facta p*ro*bar*e* valebis
Quas s*u*btraxisti p*ri*mas addendo figuras.

How to prove a subtraction sum.

¶ Here he teches þe Craft how þ*o*u schalt know, whan þ*o*u hast subt*ra*yd, wheþ*er* þou hast wel ydo or no. And þe Craft is þis, ryght as þ*o*u subtrayd þe neþ*er* figures fro þe hier figures, ryȝt so adde þe same neþ*er* figures to þe hier figures. And yf þ*o*u haue well y-wroth a-for*e* þou schalt haue þe hier nombre þe same þ*o*u haddest or þou be-gan to worch. as for þis I bade þou schulde kepe þe neþ*er* figures stylle.

Here is an example.

leaf 145 *b*. *Ensampull*e* of all*e* þe 4 cases toged*re*. worche well*e* þis case

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And yf þou worch well*e* whan þou hast all*e* subtrayd þe þ*a*t hier nombr*e* her*e*, þis schall*e* be þe nombre here foloyng whan þ*o*u hast subtrayd.

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Our author makes a slip here (3 for 1).

And þou schalt know þ*us*. adde þe neþ*er* rowe of þe same nombre to þe hier rewe as þus, cast 4 to 4. þat wol be 8. do away þe 4 & write þ*ere* 8. by þe first case of addicioɳ. þan cast 6 to 0 þat wol be 6. do away þe 0, & write þere 6. þan cast 6 to 8, þ*a*t wel be 14. do away 8 & write þ*ere* a fig*ur*e of 4, þat is þe digit, and write a fig*ur*e of 1. þ*a*t schall be-token ten. þ*a*t is þe articul vpon þe hed of 8 next aft*er*, þan reken þ*a*t 1. for 1. & cast it to 8. þat schal be 9. cast to þat 9 þe neþ*er* fig*ur*e vnd*er* þat þe quych is 4, & þat schall*e* be 13. do away þat 9 & sett þ*er*e 3, & sett a figure of 1. þ*a*t schall be 10 vpon þe next figur*is* hede þe quych is 9. by þe secu*n*de case þ*a*t þ*o*u hadest in addicioɳ. þan cast 1 to 9. & þat wol be 10. do away þe 9. & þat 1. And write þ*ere* a cifer. and write þe articull*e* þat is 1. betokenyng*e* 10. vpon þe hede of þe next figur*e* toward þe lyft side, þe quych leaf 146 *a*. *is 9, & so do forth tyl þ*o*u come to þe last 9.

He works his proof through,

take þe figure of þat 1. þe quych þ*o*u schalt fynde ou*er* þe hed of 9. & sett it ou*er* þe next figures hede þat schal be 3. ¶ Also do away þe 9. & set þ*ere* a cifer,

& þen cast þat 1 þat stondes vpon þe hede of 3 to þe same 3, & þat schalle make 4, þen caste to þe ylke 4 the figure in þe neyþer rewe, þe quych is 2, and þat schalle be 6. and brings a result.

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20004664 And þen schal þou haue an Ensampulle aȝeyn, loke & se, & but þou haue þis same þou hase myse-wroȝt.

raft of Duplation.

Sequitur de duplacione

Si vis duplare numerum, sic incipe primo
Scribe figurarum seriem quamcunque velis tu.

Four things must be known in Duplacion.

¶ This is the Chapture of duplacion, in þe quych craft þou most haue & know 4 things. ¶ Þe first þat þou most know is what is duplacion. þe secunde is how mony rewes of figures þou most haue to þis craft. ¶ þe thryde is how many cases may [6] happe in þis craft. ¶ þe fourte is what is þe profet of þe craft. ¶ As for þe first. duplacion is a doublynge of a nombre. ¶ As for þe secunde þou most leaf 146 *b.* *haue on nombre or on rewe of figures, the quych called numerus duplandus. As for þe thrid þou most know þat 3 diuerse cases may hap in þis craft. As for þe fourte. qwat is þe profet of þis craft, & þat is to know what a-risyȝt of a nombre I-doublyde. ¶ ffor-þer-more, þou most know & take gode hede in quych side þou schalle be-gyn in þis craft, or ellis þou mayst spyl alle þi laber þere aboute. certeyn þou schalt begyn in the lyft side in þis Craft. thenke wel ouer þis verse. ¶ [7]A leua dupla, diuide, multiplica. [7]

Hede the text.

Mind where you begin.

The sentens of þes verses afore, as þou may see if þou take hede. As þe text of þis verse, þat is to say, ¶ Si vis duplare. þis is þe sentence. ¶ If þou wel double a nombre þus

Remember your rules.

þou most be-gynn. Write a rewe of figures of what nomb*re* þou welt. v*er*sus.

Postea p*ro*cedas p*ri*ma*m* duplando figura*m*
Inde q*uo*d excrescit scribas vbi iusserit ordo
Iuxta p*re*cepta tibi que dant*ur* in addici*o*ne.

¶ Here he telles how þou schalt worch in þis Craft. he says, fyrst, whan þou hast writen þe nombre þou schalt be-gyn at þe first figur*e* in the lyft side, & doubull*e* þat fig*ur*e, & þe nombre þat comes þ*ere*-of þou schalt write as þou diddyst in addicion, as ¶ I schal telle þe in þe case. v*er*sus.

Cases of the Craft of Duplation.

leaf 147 *a*.

*** ¶ Nam si sit digitus in primo limite scribas.**

¶ Here is þe first case of þis craft, þe quych is þis. yf of duplacion of a figur*e* arise a digit. what schal þ*o*u do. þus þ*o*u schal do. write it in the place of the digit, do away þe fig*ur*e þat was doublede, & sett þ*ere* þe diget þat comes of þe duplacion, as þus. 23. double 2, & þ*a*t wel be 4. do away þe figur*e* of 2 & sett þ*ere* a figur*e* of 4, & so worch forth till*e* þ*o*u come to þe ende. v*er*sus.

¶ Articul*us* si sit, in p*ri*mo limite cifram,
¶ Articulu*m* v*er*o reliquis inscribe figuris;
¶ Vel p*er* se scribas, si nulla figura sequat*ur*.

¶ Here is þe secunde case, þe quych is þis yf þ*ere* come an articull*e* of þe duplacion of a fig*ur*e þ*o*u schalt do ryȝt as þ*o*u diddyst in addicion, þat is to wete þat þ*o*u schalt do away þe figur*e* þat is doublet & put a cipher in the place, and sett þ*ere* a cifer, & write þe articull*e* ou*er* þe next figur*is* hede, yf þ*ere* be any aft*er*-warde toward þe lyft side as þus. 25. begyn at the lyft side, and doubull*e* 2. þat wel be 4. do away þat 2 & sett þere 4. þan doubul 5. þat wel be 10. do away 5, & sett þ*ere* a 0, & sett 1 vpon þe next figur*is* hede þe quych is 4. & þen draw

downe 1 to 4 & þat woll*e* be 5, & þen do away þ*a*t 4 & þat 1, & sett þ*ere* 5. for þat 1 schal be rekened in þe drawyng*e* toged*re* for 1. wen leaf 147 *b.* *þou hast ydon þou schalt haue þis nomb*re* 50.

If there is no figure to ‘carry’ them to, write them down.

yf þ*ere* come no figur*e* aft*er* þe fig*ur*e þ*a*t is addit, of þe quych addicioꝺ comes an articull*e*, þ*o*u schalt do away þe figur*e* þ*a*t is dowblet & sett þ*ere* a 0. & write þe articul next by in þe same rewe toward þe lyft syde as þus, 523. double 5 þat woll be ten. do away þe figur*e* 5 & set þ*ere* a cifer, & sett þe articul next aft*er* in þe same rewe toward þe lyft side, & þou schalt haue þis nombre 1023. þen go forth & double þe oþ*er* nombers þe quych is ly3t y-now3t to do. v*er*sus.

¶ Compositus si sit, in limite sc*ri*be seq*uen*te
Articulu*m*, p*ri*mo digitu*m*; q*uia* sic iubet ordo:
Et sic de reliq*ui*s facie*n*s, si sint tibi plures.

If it is a Composite, write down the digit, and ‘carry’ the tens.

¶ Here he putt*es* þe Thryd case, þe quych is þis, yf of duplacioꝺ of a fig*ur*e come a Composit. þ*o*u schalt do away þe fig*ur*e þ*a*t is doublet & set þ*ere* a digit of þe Composit, & sett þe articull*e* ou*er* þe next figures hede, & aft*er* draw hym downe w*ith* þe figur*e* ou*er* whos hede he stondes, & make þ*ere*-of an nombre as þ*o*u hast done afore, & yf þ*ere* come no fig*ur*e aft*er* þat digit þat þ*o*u hast y-write, þa*n* set þe articull*e* next aft*er* hym in þe same rewe as þus, 67: double 6 þat wel be 12, do away 6 & write þ*ere* þe digit leaf 148 *a.* *of 12, þe quych is 2,

Here is an example.

and set þe articull*e* next aft*er* toward þe lyft side in þe same rewe, for þ*ere* comes no figur*e* aft*er*. þan dowble þat oþ*er* figur*e*, þe quych is 7, þat wel be 14. the quych is a Composit. þen do away 7 þat þ*o*u doublet & sett þe þe diget of hy*m*, the quych is 4, sett þe articull*e* ou*er* þe next figur*es* hed, þe quych is 2, & þen draw to hym þat on, & make on nombre þe quych schall*e* be 3. And þen yf þ*o*u haue wel y-do þ*o*u schall*e* haue þis nombre of þe duplacioꝺ, 134. v*er*sus.

¶ Si super ext*re*ma*m* nota sit monade*m* dat eid*em*

Quod t*ibi con*tingat si p*ri*mo dimidiabis.

How to double the mark for one-half.

¶ Here he says, yf ou*er* þe fyrst fig*ur*e in þe ryȝt side be such a merke as is her*e* made, ^{w}, þ*o*u schall*e* fyrst doubull*e* þe figur*e*, the quych stondes vnd*er* þ*a*t merke, & þen þou schalt doubul þat merke þe quych stond*es* for haluendel on. for too haluedels makes on, & so þ*a*t wol be on. cast þ*a*t on to þat duplacioɳ of þe figur*e* ou*er* whos hed stode þat merke, & write it in þe same place þ*ere* þat þe figur*e* þe quych was doublet stode, as þus 23^{w}. double 3, þat wol be 6; doubul þat halue on, & þat wol be on. cast on to 6, þ*a*t wel be 7. do away 6 & þat 1, & sett þ*ere* 7. þan hase þou do. as for þat figur*e*, þan go leaf 148 *b*. *to þe oþ*er* fig*ure* & worch forth.

This can only stand over the first figure.

& þou schall neu*er* haue such a merk but ou*er* þe hed of þe furst figure in þe ryght side. And ȝet it schal not happe but yf it were y-halued a-for*e*, þus þ*o*u schalt vnd*er*stonde þe verse. ¶ Si sup*er* ext*r*ema*m* &c. Et nota, talis fig*ur*a ^{w} significans medietate*m*, unitat*is* veniat, *i.e.* contingat u*e*l fiat sup*er* ext*r*ema*m*, *i.e.* sup*er* p*r*ima*m* figura*m* in ext*r*emo sic v*er*sus dextram ars dat: *i.e.* reddit monade*m*. *i.e.* vnitate*m* eide*m*. *i.e.* eidem note & declina*tur* hec monos, d*i*s, di, dem, &c. ¶ Quod *er*go to*tum* ho*c* dabis monade*m* note *con*ting*et*. *i.e.* eveniet tibi si dimidiasti, *i.e.* accipisti u*e*l subtulisti medietatem alicuius unius, in cuius principio sint figura nu*mer*u*m* denotans i*m*pare*m* p*rim*o *i.e.* principiis.

Craft of Mediation.

¶ Sequit*ur* de mediacione.

Incipe sic, si vis alique*m* nu*mer*u*m* mediar*e*:
Sc*ri*be figurar*um* seriem sola*m*, velut an*te*.

The four things to be known in this chapter

¶ In þis Chapter is taȝt þe Craft of mediacioun, in þe quych craft þou most know 4 thynges. ffurst what is mediacion. the secunde how mony rewes of figures þou most haue in þe wyrchynge of þis craft. þe thryde how mony diuerse cases may happ in þis craft. [8] the for þe furst, þou schalt vndurstonde þat mediacion is a takyng out of halfe a nomber out of a holle nomber, leaf 149 *a*. *as yf þou wolde take 3 out of 6. ¶ As for þe secunde, þou schalt know þat the second, þou most haue one rewe of figures, & no moo, as þou hayst in craft of duplacion. ¶ As for the thryd, þou most vnderstonde the third, þat cases may happe in þis craft. ¶ As for þe fourte, þou schalle know þat the profet of þis craft is when þou hast take away þe haluendel of a nombre to telle qwat þere schalle leue. ¶ Incipe sic, &c. The sentence of þis verse is þis. yf þou wold medye, þat is to say, take halfe out of þe holle, or halfe out of halfe, þou most begynne þe first rewe of figures of what nombre þou wolte, as þou dyddyst be-fore in þe Craft of duplacion. versus.

Write one rewe

Begynne

¶ Postea procedas medians, si prima figura
Si par aut impar videas.

¶ Here he says, when þou hast write a rewe of figures, þou schalt take-hede whether þe first figure be euen or odde in nombre, & vnderstonde þat he spekes of þe first figure in þe ryȝt side. And in the ryght side þou schalle begynne in þis Craft.

See if the number is even or odd

¶ Quia si fuerit par,
Dimidiabis eam, scribens quicquid remanebit:

If it is even, halve it, and write the answer in its place

¶ Here is the first case of þis craft, þe quych is þis, yf þe first figure be euen. þou schal take away fro þe figure euen halfe, & do away þat figure and set þere þat leues ouer, as þus, 4. take leaf 149 *b*. *halfe out of 4, & þan þere leues 2. do away 4 & sett þere 2. þis is lyght y-nowȝt. versus.

Mediation of an Odd Number.

¶ Impar si fu*er*it vnu*m* demas mediar*e*
Quod no*n* p*re*sumas, s*ed* quod sup*er*est mediabis
Inde sup*er* tractu*m* fac demptu*m* quod no*ta*t vnu*m*.

If it is odd, halve the even number less than it.

Here is þe secunde case of þis craft, the quych is þis. yf þe first figur*e* betoken*e* a nombre þat is odde, the quych odde schal not be mediete, þen þ*o*u schalt medye þat nombre þat leues, when the odde of þe same nomb*re* is take away, & write þat þ*a*t leues as þ*o*u diddest in þe first case of þis craft. Whaŋ þ*o*u hayst write þat. for þ*a*t þat leues,

Then write the sign for one-half over it.

write such a merke as is her*e* ^{w} vpon his hede, þe quych merke schal betokeŋ halfe of þe odde þat was take away.

Here is an example.

lo an Ensampull. 245. the first figur*e* her*e* is betokenyng*e* odde nombre, þe quych is 5, for 5 is odde; þ*er*e-for*e* do away þat þ*a*t is odde, þe quych is 1. þen leues 4. þen medye 4 & þen leues 2. do away 4. & sette þ*ere* 2, & make such a merke ^{w} upon his hede, þat is to say ou*er* his hede of 2 as þus. 242.^{w} And þen worch forth in þe oþ*er* figures tyll þ*o*u come to þe ende. by þe furst case as þ*o*u schalt vnd*er*stonde þat

Put the mark only over the first figure.

þou schalt Leaf 150 *a.* *neu*er* make such a merk but ou*er* þe first fig*ur*e hed in þe riȝt side. Wheþ*er* þe other fig*ur*es þat comyŋ aft*er* hym be eueŋ or odde. v*er*sus.

Cases of the Craft of Mediation.

¶ Si monos, dele; sit t*ibi* cifra post no*ta* supra.

If the first figure is one put a cipher.

¶ Here is þe thryde case, þe quych yf the first figur*e* be a figur*e* of 1. þ*o*u schalt do away þat 1 & set þ*ere* a cifer, & a merke ou*er* þe cifer as þus, 241. do away 1, & sett þ*ere* a cifer w*ith* a merke ou*er* his hede, & þen hast þ*o*u ydo for þat 0. as þus 0^{w} þen worch forth in þe oþer fig*ur*ys till

þ*o*u come to þe ende, for it is lyght as dyche water. vn*de* ve*r*sus.

¶ Postea p*ro*cedas hac condic*i*one secu*n*da:
Imp*ar* si fu*er*it hinc vnu*m* deme p*ri*ori,
Inscribens quinque, nam denos significabit
Monos p*re*d*ict*am.

What to do if any other figure is odd.

¶ Here he putt*es* þe fourte case, þe quych is þis. yf it happeŋ the secunde figur*e* betoken odde nombre, þou schal do away on of þat odde nombre, þe quych is significatiue by þ*a*t figure 1. þe quych 1 schall be rekende for 10. Whan þ*o*u hast take away þ*a*t 1 out of þe nombre þ*a*t is signifiede by þat figur*e*, þ*o*u schalt medie þ*a*t þat leues ou*er*, & do away þat figur*e* þat is medied, & sette in his styde halfe of þ*a*t nombre.

Write a figure of five over the next lower number's head.

¶ Whan þou hase so done, þ*o*u schalt write leaf 150 *b*. *a figure of 5 ou*er* þe next figur*es* hede by-for*e* toward þe ryȝt side, for þat 1, þe quych made odd nombr*e*, schall stonde for ten, & 5 is halfe of 10; so þ*o*u most write 5 for his haluendell*e*.

Example.

lo an Ensampull*e*, 4678. begyŋ in þe ryȝt side as þ*o*u most nedes. medie 8. þen þ*o*u schalt leue 4. do away þat 8 & sette þ*ere* 4. þen out of 7. take away 1. þe quych makes odde, & sett 5. vpon þe next figur*es* hede afor*e* toward þe ryȝt side, þe quych is now 4. but afor*e* it was 8. for þat 1 schal be rekenet for 10, of þe quych 10, 5 is halfe, as þou knowest wel. Whan þ*o*u hast þus ydo, medye þ*a*t þe quych leues aft*er* þe takyng*e* away of þat þat is odde, þe quych leuyng*e* schall*e* be 3;

5
4634. do away 6 & sette þ*er*e 3, & þou schalt haue such a nombre aft*er* go forth to þe next fig*ur*e, & medy þat, & worch forth, for it is lyȝt ynovȝt to þe c*er*tayŋ.

¶ Si v*er*o s*ecun*da dat vnu*m*.
Illa deleta, sc*ri*bat*ur* cifra; p*ri*ori
¶ Tradendo quinque pro denario mediato;
Nec cifra sc*ri*batur, nisi dei*n*de fig*ur*a seq*u*at*ur*:

Postea p*ro*cedas reliq*ua*s mediando figuras
Vt sup*ra* docui, si sint tibi mille figure.

If the second figure is one, put a cipher, and write five over the next figure.

¶ Her*e* he putt*es* þe 5 case, þe quych is leaf 151 *a*. *þis: yf þe secunde figur*e* be of 1, as þis is here 12, þou schalt do away þat 1 & sett þ*ere* a cifer. & sett 5 ou*er* þe next fig*ur*e hede afor*e* toward þe riȝt side, as þou diddyst afor*e*; & þat 5 schal be haldel of þat 1, þe quych 1 is rekent for 10. lo an Ensampull*e*, 214. medye 4. þ*a*t schall*e* be 2. do away 4 & sett þ*ere* 2. þe*n* go forth to þe next figur*e*. þe quych is bot 1. do away þat 1. & sett þ*ere* a cifer. & set 5 vpon þe figur*es* hed afor*e*, þe quych is nowe 2, & þen þou schalt haue þis no*m*br*e* 5
202, þen worch forth to þe nex fig*ur*e. And also it is no mayst*er*y yf þ*ere* come no figur*e* after þat on is medyet, þ*o*u schalt write no 0. ne nowȝt ellis, but set 5 ou*er* þe next fig*ur*e afor*e* toward þe ryȝt, as þus 14.

How to halve fourteen.

medie 4 then leues 2, do away 4 & sett þ*ere* 2. þen medie 1. þe q*ui*ch is rekende for ten, þe halue*n*del þ*ere*-of wel be 5. sett þ*a*t 5 vpon þe hede of þ*a*t figur*e*, þe quych is now 2, 5
2, & do away þ*a*t 1, & þou schalt haue þis nombre yf þ*o*u worch wel, vn*de* ver*s*us.

to prove the Mediation.

¶ Si mediacio sit b*ene* f*ac*ta p*ro*bar*e* valeb*is*
¶ Duplando num*eru*m que*m* p*ri*mo di*m*ediasti

How to prove your mediation.

¶ Her*e* he telles þe how þou schalt know wheþ*er* þou hase wel ydo or no. doubul leaf 151 *b*. *þe nombre þe quych þ*o*u hase mediet, and yf þ*o*u haue wel y-medyt after þe dupleacioɳ, þou schalt haue þe same nombre þat þ*o*u haddyst in þe tabull*e* or þ*o*u began to medye, as þus.

First Example.

[The furst ensampull*e* was þis. 4. þe quych I-mediet was laft 2, þe whych 2 was write in þe place þ*a*t 4 was write afor*e*. Now doubull*e* þat 2, & þ*o*u schal haue 4, as þ*o*u hadyst afor*e*.

The second.

þe secunde Ensampull*e* was þis, 245. When þ*o*u haddyst mediet

all*e* þis nomb*re*, yf þou haue wel ydo þou schalt haue of þ*a*t mediacion̨ þis nombre, 122^{w}. Now doubull*e* þis nombre, & begyn in þe lyft side; doubull*e* 1, þat schal be 2. do away þat 1 & sett þ*ere* 2. þen doubull*e* þ*a*t oþ*er* 2 & sett þ*ere* 4, þen doubull*e* þat oþ*er* 2, & þat wel be 4. þe*n* doubul þat merke þat stondes for halue on. & þat schall*e* be 1. Cast þat on to 4, & it schall*e* be 5. do away þat 2 & þat merke, & sette þ*ere* 5, & þen þ*o*u schal haue þis nombre 245. & þis wos þe same nombur þ*a*t þ*o*u haddyst or þ*o*u began to medye, as þ*o*u mayst se yf þou take hede. The nombre þe quych þou haddist for an Ensampul in þe 3 case of mediacion̨ to be mediet was þis 241. whan þ*o*u haddist medied all*e* þis nombur truly leaf 152 *a.* *by eu*er*y figur*e*, þou schall haue be þ*a*t mediacion̨ þis nombur 120^{w}. Now dowbul þis nomb*ur*, & begyn in þe lyft side, as I tolde þe in þe Craft of duplacion̨. þus doubull*e* þe fig*ur*e of 1, þat wel be 2. do away þat 1 & sett þ*ere* 2, þen doubul þe next figur*e* afore, the quych is 2, & þat wel be 4; do away 2 & set þ*ere* 4. þen doubul þe cifer, & þat wel be no3t, for a 0 is no3t. And twyes no3t is but no3t. þ*ere*fo*r*e doubul the merke aboue þe cifers hede, þe quych betokenes þe halue*n*del of 1, & þat schal be 1. do away þe cifer & þe merke, & sett þ*ere* 1, & þen þ*o*u schalt haue þis nombur 241. And þis same nombur þ*o*u haddyst afore or þ*o*u began to medy, & yf þ*o*u take gode hede. ¶ The next ensampul þat had in þe 4 case of mediacion̨ was þis 4678. Whan þ*o*u hast truly ymedit all*e* þis nombur fro þe begynnyng*e* to þe endyng*e*, þ*o*u schalt haue of þe mediacion̨ þis nombur 5
2334. Now doubul this nombur & begyn in þe lyft side, & doubull*e* 2 þat schal be 4. do away 2 and sette þere 4; þen doubul*e* 3, þ*a*t wol be 6; do away 3 & sett þ*ere* 6, þen doubul þat oþ*er* 3, & þat wel be 6; do away 3 & set þ*ere* leaf 152 *b.* *6, þen doubul þe 4, þat welle be 8; þen doubul 5. þe quych stondes ou*er* þe hed of 4, & þat wol be 10; cast 10 to 8, & þ*a*t schal be 18; do away 4 & þat 5, & sett þ*ere* 8, &

sett that 1, þe quych is an articul of þe Composit þe quych is 18, ou*er* þe next figur*es* hed toward þe lyft side, þe quych is 6. drav þ*a*t 1 to 6, þe quych 1 in þe dravyng schal be rekente bot for 1, & þ*a*t 1 & þ*a*t 6 togedur wel be 7. do away þat 6 & þat 1. the quych stondes ou*er* his hede, & sett ther 7, & þen þou schalt haue þis nombur 4678. And þis same nombur þ*o*u hadyst or þ*o*u began to medye, as þ*o*u mayst see in þe secunde Ensampul þat þou had in þe 4 case of mediacioɳ, þat was þis: when þou had mediet truly all*e* the nombur, a p*ri*ncipio usque ad fine*m*. þ*o*u schalt haue of þat mediacioɳ þis nombur 5

The fifth example.

102. Now doubul 1. þat wel be 2. do away 1 & sett þ*ere* 2. þen doubul 0. þ*a*t will be noȝt. þ*ere*fo*r*e take þe 5, þe quych stondes ou*er* þe next figur*es* hed, & doubul it, & þat wol be 10. do away þe 0 þat stondes betwene þe two fig*uri*s, & sette þ*ere* in his stid 1, for þ*a*t 1 now schal stonde in þe secunde place, wher*e* he schal betoken 10; þen doubul 2, þat wol be 4. do away 2 & sett þere 4. & leaf 153 *a.* *þou schal haue þus nombur 214. þis is þe same nu*m*bur þat þ*o*u hadyst or þ*o*u began to medye, as þ*o*u may see. And so do eu*er* mor*e*, yf þ*o*u wil knowe whe þ*er* þou hase wel ymedyt or no.

¶. doubull*e* þe nu*m*bur þat comes aft*er* þe mediaciouɳ, & þ*o*u schal haue þe same nombur þ*a*t þ*o*u hadyst or þ*o*u began to medye, yf þ*o*u haue welle ydo. or els doute þe noȝt, but yf þ*o*u haue þe same, þ*o*u hase faylide in þ*i* Craft.

raft of Multiplication.

Sequitur de multiplicatione.

ite down a Multiplication Sum.

S i tu p*er* num*er*u*m* num*er*u*m* vis m*u*ltiplicar*e*

Scribe duas q*ua*scu*nque* velis series nu*me*ror*um*

Ordo s*er*vet*ur* vt vltima m*u*ltiplicandi

Ponat*ur* sup*er* ant*er*iorem multiplicant*is*
A leua reliq*u*e sint scripte m*u*ltiplicantes.

Four things to be known of Multiplicacion.

¶ Here begynnes þe Chapt*r*e of m*u*ltiplicaciọn, in þe quych þou most know 4 thynges. ¶ Ffirst, qwat is m*u*ltiplicaciọn. The secunde, how mony cases may hap in multiplicaciọn. The thryde, how mony rewes of figur*es* þ*er*e most be. ¶ The 4. what is þe p*ro*fet of þis craft.

The first.

¶ As for þe first, þ*o*u schal vnd*er*stonde þat m*u*ltiplicaciọn is a bryngynge to-ged*er* of 2 thynges in on nombur, þe quych on nombur *con*tynes so mony tymes on, howe leaf 153 *b.* *mony tymes þ*er*e ben vnytees in þe nowmb*re* of þat 2, as twyes 4 is 8. now her*e* ben þe 2 nomb*er*s, of þe quych too nowmbres on is betokened be an adu*er*be, þe quych is þe worde twyes, & þis worde thryes, & þis worde four*e* sythes,[9] & so furth of such other lyke wordes. ¶ And tweyn nombres schal be tokenyde be a nowne, as þis worde four*e* showys þes tweyn nombres y-broth in-to on hole nombur, þat is 8, for twyes 4 is 8, as þ*o*u wost wel. ¶ And þes nomb*re* 8 conteynes as oft tymes 4 as þ*er*e ben vnites in þ*a*t other nomb*re*, þe quych is 2, for in 2 ben 2 vnites, & so oft tymes 4 ben in 8, as þ*o*u wottys wel.

The second.

¶ ffor þe secund*e*, þ*o*u most know þat þ*o*u most haue too rewes of figures.

The third.

¶ As for þe thryde, þ*o*u most know þ*a*t 8 man*er* of diu*er*se case may happe in þis craft.

The profet of þis Craft.

¶ The p*ro*fet of þis Craft is to telle when a nomb*re* is m*u*ltiplyed be a noþ*er*, qwat co*m*mys þ*er*e of. ¶ fforthermor*e*, as to þe sentence of our*e* verse, yf þ*o*u wel m*u*ltiply a nombur be a-noþ*er* nomb*ur*, þou schalt write leaf 154 *a.* *a rewe of figures of what nomb*ur*s so eu*er* þ*o*u welt, & þat schal be called Num*erus* m*u*ltiplicand*us*, Anglice, þe nomb*ur* the quych to be m*u*ltiplied. þen þ*o*u schalt write a-nother rewe of figur*es*, by þe quych þ*o*u schalt m*u*ltiplie the nombre þat is to be m*u*ltiplied, of þe quych nomb*ur* þe furst fig*ur*e schal be write vnd*er* þe last figur*e* of þe nomb*ur*, þe quych is to be m*u*ltiplied.

How to set down the sum.

And so write forthe toward þe lyft side, as her*e* you may se,

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1234 And þis one nombur schalle be called numerus multiplicans. Anglice, þe nombur multipliynge, for he schalle multiply þe hyer nounbur, as þus one tyme 6. And so forth, as I schal telle the afterwarde. And þou schal begyn in þe lyft side. Two sorts of Multiplication treated of. ¶ ffor þere is two more þou schalt vndurstonde þat þere is two manurs of multiplicacion; one ys of þe wyrchynge of þe boke only in þe mynde of a mon. fyrst he teches of þe fyrst maner of duplacion, þe quych is be wyrchynge of tabul. Afterwarde he wol teche on þe secunde maner. vnde versus. And so on page.

ultiply one Digit by another.

In digitum cures digitum si ducere maior
leaf 154 *b*.
*** Per quantum distat a denis respice debes**
¶ Namque suo decuplo totiens delere minorem
Sitque tibi numerus veniens exinde patebit.

How to multiply two digits. ¶ Here he teches a rewle, how þou schalt fynde þe nounbre þat comes by þe multiplicacion of a digit be anoþer. loke how mony [vny]tes ben. bytwene þe more digit and 10. And reken ten for on vnite. Subtract the greater from ten, And so of de away þe lasse nounbre out of his owne decuple, þat is to say, fro þat nounbre þat is ten tymes so mych is þe nounbre þat comes of þe multiplicacion. As yf þou wol multiply 2 be 4. loke how mony vnitees ben by-twene þe quych is þe more nounbre, & be-twene ten. Certen þere wel be vj vnitees by-twene 4 & ten. yf þou reken þere with þe ten þe vnite, as þou may se. take the less so many times from ten &c. so mony tymes take 2. out of his decuple, þe quych is 20. for 20 is þe decuple of 2, 10 is þe decuple of 1, 30 is þe decuple of 3, 40 is þe decuple of 4, And þe oþer digetes til þou come to ten; & whan þou hast y-take so mony tymes 2 out of twenty, þe quych is sex tymes, Example. þou schal leue 8 as þou wost wel, for 6 times 2 is twelue. take [1]2 out of twenty, & þere schal leue 8. bot yf bothe þe digettes leaf 155 *a*. *ben y-lyech mych as here. 222 or too

tymes twenty, þen it is no fors quych of hem tweyn þou take out of here decuple. als mony tymes as þat is fro 10. but neu*er*-þe-lesse, yf þou haue hast to worch, þou schalt haue here a tabul of figures, where-by þou schalt se a-nonn ryght what is þe nounbre þat comes of þe multiplicacion of 2 digittes. þus þou schalt worch in þis fig*ur*e.

Better use this table, though

1									
2	4								
3	6	9							
4	8	12	16						
5	10	15	20	25					
6	12	18	24	30	36				
7	14	21	28	35	42	49			
8	16	24	32	40	48	56	64		
9	18	27	36	45	54	63	72	81	
1	2	3	4	5	6	7	8	9	

yf þe figu*r*e þe quych schall*e* be m*u*ltiplied, be euen*e* as mych as þe diget be, þe quych þat oþ*er* figur*e* schal be m*u*ltiplied, as two tymes twayn, or thre tymes 3. or sych other. The way to use the Multiplication table

loke qwere þat fig*ur*e sittes in þe lyft side of þe t*ri*angle, & loke qwer*e* þe diget sittes in þe neþ*er* most rewe of þe triangle. & go fro hym vpwarde in þe same rewe, þe quych rewe gose vpwarde til þou come agaynes þe oþ*er* digette þat sittes in þe lyft side of þe t*ri*angle. And þat nounbre, þe quych þou leaf 155 *b.* fyn*des þ*ere* is þe nounbre þat comes of the m*u*ltiplicacion of þe 2 digittes, as yf þou wold wete qwat is 2 tymes 2. loke quer*e* sittes 2 in þe lyft side i*n* þe first rewe, he sittes next 1 in þe lyft side al on hye, as þou may se; þe[*n*] loke qwer*e* sittes 2 in þe lowyst rewe of þe t*ri*angle, & go fro hym vpwarde in þe same rewe tyll*e* þou come a-ȝenenes 2 in þe hyer place, & þer þou schalt fynd ywrite 4, & þat is þe nounb*r*e þat comes of þe multiplicacion of two tymes tweyn is 4, as þow wotest well*e*. yf þe diget. the quych is m*u*ltiplied, be mor*e* þan þe

oþ*er*, þou schalt loke qwer*e* þe mor*e* diget sittes in þe lowest rewe of þe t*ri*angle, & go vpwarde in þe same rewe tyl [10] þ*o*u come a-nendes þe lasse diget in the lyft side. And þ*ere* þ*o*u schalt fynde þe no*m*br*e* þat comes of þe m*u*ltiplicacioɳ; but þ*o*u schalt vnd*er*stonde þat þis rewle, þe quych is in þis v*er*se. ¶ In digitu*m* cures, &c., noþ*er* þis t*ri*angle schall*e* not s*er*ue, bot to fynde þe nounbres þ*a*t comes of the m*u*ltiplicacioɳ þat comes of 2 articuls or *com*posites, þe nedes no craft but yf þou wolt m*u*ltiply in þi mynde. And leaf 156 *a.* *þere-to þou schalt haue a craft aft*er*warde, for þou schall wyrch w*ith* digettes in þe tables, as þou schalt know aft*er*warde. v*er*sus.

ltiply one Composite by another.

¶ Postea p*ro*cedas postrema*m* m*u*ltiplica*n*do
[Recte multiplicans per cu*n*ctas i*n*feriores]
Condic*i*onem tamen t*a*li q*uod* m*u*ltiplicant*es*
Scribas in capite quicq*ui*d p*ro*cesserit inde
Sed postq*uam* fuit hec m*u*ltiplicate fig*ur*e
Anteriorent*ur* serei m*u*ltiplica*n*t*is*
Et sic m*u*ltiplica velut isti m*u*ltiplicasti
Qui sequit*ur* nu*mer*u*m* scr*i*ptu*m* quiscu*n*q*ue* figur*is*.

How to multiply one number by another.

¶ Here he teches how þou schalt wyrch in þis craft. þou schalt m*ul*tiplye þe last figur*e* of þe nombre, and quen þ*o*u hast so ydo þou schalt draw all*e* þe figures of þe neþ*er* nounbre mor*e* taward þe ryȝt side, so qwe*n* þ*o*u hast m*u*ltiplyed þe last figur*e* of þe heyer nounbre by all*e* þe neþ*er* figures.

Multiply the 'last' figure of the higher by the 'first' of the lower number.

And sette þe nounbir þat comes þer-of ou*er* þe last figur*e* of þe neþ*er* nounb*re*, & þen þou schalt sette al þe oþ*er* fig*ur*es of þe neþ*er* nounb*re* mor*e* ner*e* to þe ryȝt side. ¶ And whan þou hast m*u*ltiplied þat figur*e* þat schal be m*u*ltiplied þe next aft*er* hym by al þe neþ*er* figures. And worch as þou dyddyst afor*e* til leaf 156 *b.* *þou come to þe ende. And þou schalt vnd*er*stonde þat eu*er*y figur*e* of þe hier nounb*re* schal be

m*u*ltiplied be all*e* þe figur*e*s of the neþ*er* nounbre, yf þe hier nounb*re* be any figur*e* þen on*e*. Set the answer over the first of the lower
an Ensample here folowynge. 2465.
232 þou schalt begyne to m*u*ltiplye in þe lyft side. M*u*ltiply 2 be 2, and twyes 2 is 4. set 4 then multiply the second of the lower,
ouer þe hed of þat 2, þen m*u*ltiplie þe same hier 2 by 3 of þe nether nounbre, as thryes 2 þat schal be 6. set 6 ou*er* þe hed of 3, þan m*u*ltiplie þe same hier 2 by þat 2 þe quych stondes vnd*er* hym, þ*a*t wol be 4; do away þe hier 2 & sette þ*ere* 4.

Then antery the lowe nounbre ¶ Now þou most antery þe nether nounbre, þat is to say, þ*o*u most sett þe neþ*er* nounbre more towarde þe ryȝt side, as þus. Take þe neþ*er* 2 toward þe ryȝt side, & sette it euen vnd*er* þe 4 of þe hyer nounb*r*e, & ant*er*y all*e* þe figures þat comes aft*er* þat 2, as þus; sette 2 vnd*er* þe 4. þen sett þe figur*e* of 3 þ*ere* þat þe figure of 2 stode, þe quych is now vndur þ*a*t 4 in þe hier nounbre; þen sett þe oþer figur*e* of 2, þe quych is þe last fig*ur*e toward þe lyft side of þe neþ*er* nomb*er* þ*ere* þe figur*e* of 3 stode. þou schalt haue such a nombre. 464465

232 leaf 157 *a*. * ¶ Now m*u*ltiply 4, þe quych comes next aft*er* 6, by þe last 2 of þe neþ*er* nounbur toward þe lyft side. as 2 tymes 4, þat wel be 8. sette þat 8 ou*er* þe figure the quych stondes ou*er* þe hede of þat 2, þe quych is þe last figur*e* of þe neþ*er* nounbre; þan multiplie þat same 4 by 3, þat comes in þe neþ*er* rewe, þat wol be 12. sette þe digit of þe composyt ou*er* þe figure þe quych stondes ou*er* þe hed of þat 3, & sette þe articule of þis co*m*posit ou*er* al þe figures þat stondes ou*er* þe neþ*er* 2 hede. Now multiply by the last but one

þen multiplie þe same 4 by þe 2 in þe ryȝt side in þe neþ*er* nounbur, þat wol be 8. do away 4. & sette þ*ere* 8. Eu*er* mor*e* qwen þ*o*u m*u*ltiplies þe hier figur*e* by þat figur*e* þe quych stondes vnd*er* hym, þou schalt do away þat hier figur*e*, & sett þer þat nounbre þe quych comes of m*u*ltiplicacion of ylke dygitus. Whan þou hast done as I haue byde þe, þ*o*u schalt haue suych an ord*er* of figur*e* as is her*e*, 1

4648[65]

232. þen take and ant*er*y þi neþ*er* figures. And sett þe fyrst fig*ur*e of þe neþ*er* figures [11] vndre be figur*e* of 6. ¶ And draw al þe oþ*er* figures of þe same rewe to hym-warde, leaf 157 *b.* *as þ*o*u diddyst afore. þen m*u*ltiplye 6 be 2, & sett þat þe quych comes ou*er* þ*ere*-of ou*er* al þe oþ*er* figures hedes þat stondes ou*er* þat 2. þen m*u*ltiply 6 be 3, & sett all*e* þat comes þ*ere*-of vpon all*e* þe figur*e*s hedes þat standes ou*er* þat 3; þa*n* m*u*ltiplye 6 be 2, þe quych stondes vnd*er* þat 6, þen do away 6 & write þ*ere* þe digitt of þe composit þat schal come þ*ere*of, & sette þe articull ou*er* all*e* þe figures þat stondes ou*er* þe hede of þat 3 as here, 11

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232 Antery the figures again, and multiply by five. þen antery þi figures as þou diddyst afor*e*, and m*u*ltipli 5 be 2, þat wol be 10; sett þe 0 ou*er* all þe figures þ*a*t stonden ou*er* þat 2, & sett þ*a*t 1. ou*er* the next figures hedes, all*e* on hye towarde þe lyft side. þen m*u*ltiplye 5 be 3. þat wol be 15, write 5 ou*er* þe figures hedes þat stonden ou*er* þ*a*t 3, & sett þat 1 ou*er* þe next figur*e*s hedes toward þe lyft side. þen m*u*ltiplye 5 be 2, þat wol be 10. do away þat 5 & sett þ*ere* a 0, & sett þat 1 ou*er* þe figures hedes þat stonden ou*er* 3. And þen leaf 158 *a.* þou schalt haue such a nounbre as here stondes aftur.* 11

1101

1215

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232 ¶ Now draw all*e* þese figures downe toged*er* as þus, 6.8.1. & 1 draw to-gedur; þat wolle be 16, do away all*e* þese figures saue 6. lat hym stonde, for þow þ*o*u take hym away þou most write þer þe same a3ene. þ*ere*for*e* late hym stonde, & sett 1 ou*er* þe figur*e* hede of 4 toward þe lyft side; þen draw on to 4, þat woll*e* be 5. Then add all the figures above the line: do away þat 4 & þat 1, & sette þ*ere* 5. þen draw 4221 & 1 toged*ur*, þat wol

be 10. do away all*e* þat, & write þere þat 4 & þat 0, & sett þat 1 ou*er* þe next figur*es* hede toward þe lyft side, þe quych is 6. þen draw þat 6 & þat 1 togedur, & þat wolle be 7; do away 6 & sett þ*ere* 7, þen draw 8810 & 1, & þat wel be 18; do away all*e* þe figures þ*a*t stondes ou*er* þe hede of þat 8, & lette 8 stonde stil, & write þat 1 ou*er* þe next fig*ur*i*s* hede, þe quych is a 0. þen do away þat 0, & sett þ*ere* 1, þe quych stondes ou*er* þe 0. hede. þen draw 2, 5, & 1 toged*ur*, þat woll*e* be 8. þen do away all*e* þat, & write þ*ere* 8. and you will have the summ.

¶ And þen þou schalt haue þis nounbre, 571880.

Cases of this Craft.

leaf 158 *b*.

*** ¶ S*ed* cu*m* m*u*ltiplicabis, p*ri*mo sic e*st* op*er*andu*m*,**
Si dabit articulu*m* tibi m*u*ltiplicacio solu*m*;
P*ro*posita cifra su*m*ma*m* t*ra*nsferre memen*to*.

What to do if the first multiplication results in an article. ¶ Here he puttes þe fyrst case of þis craft, þe quych is þis: yf þ*ere* come an articulle of þe m*u*ltiplicacion̨ ysette befor*e* the articull*e* in þe lyft side as þus 51
23. multiplye 5 by 2, þat wol be 10; sette ou*er* þe hede of þat 2 a 0, & sett þat on, þat is þe articul, in þe lyft side, þat is next hym, þen þ*o*u schalt haue þis nounbre 1051.
23 ¶ And þen worch forth as þou diddist afore. And þ*o*u schalt vnd*er*stonde þat þ*o*u schalt write no 0. but whan þat place where þou schal write þat 0 has no figure afore hy*m* noþ*er* aft*er*. v*er*sus.

¶ Si aut*em* digitus excreu*er*it articul*us*q*ue*.
Articul*us*[12] sup*ra*p*osit*o digito salit vltra.

What to do if the result is a composite number. ¶ Here is þe secunde case, þe quych is þis: yf hit happe þat þ*ere* come a composyt, þou schalt write þe digitte ou*er* þe hede of þe neþ*er* figur*e* by þe quych þ*o*u multipliest þe hier figure; and sett þe articull*e* next hym toward þe lyft side, as þou diddyst afore, as þ*us* 83.
83 Multiply 8 by 8, þat wol be 64. Write þe 4 ou*er* 8, þat is to say, ou*er* þe hede of þe neþ*er* 8; & set 6, þe quych leaf 159 *a.* *is an articul, next aft*er*. And þen þou schalt haue such a nounb*r*e as is here, 6483[13],
83 And þen worch forth.

¶ Si digitus t*amen* ponas ip*su*m sup*er* ipsam.

What it is a digit. ¶ Here is þe thryde case, þe quych is þis: yf hit happe þat of þi m*u*ltiplicaciou̯n come a digit, þ*o*u schalt write þe digit ou*er* þe hede of þe neþ*er* figur*e*, by the quych þou m*u*ltipliest þe hier*e* figur*e*, for þis nedes no Ensampul.

¶ Subdita m*u*ltiplica non hanc que [incidit] illi
Delet ea*m* penit*us* scribens quod p*ro*uenit inde.

The fourth chapitre of the craft. ¶ Here is þe 4 case, þe quych is: yf hit be happe þat þe neþ*er* figur*e* schal multiplye þat figur*e*, þe quych stondes ou*er* þat figures hede, þou schal do away þe hier figur*e* & sett þ*er*e þat þ*a*t comys of þ*a*t m*u*ltiplicacioɳ. As yf þ*er*e come of þat m*u*ltiplicacioɳ an articuls þou schalt write þere þe hier figur*e* stode a 0. ¶ And write þe articuls in þe lyft side, yf þat hit be a digit write þ*er*e a digit. yf þat h*i*t be a composit, write þe digit of þe composit. And þe articul in þe lyft side. al þis is ly3t y-now3t, þ*er*e-for*e* þer nedes no Ensampul.

¶ S*ed* si m*u*ltiplicat alia*m* ponas sup*er* ipsam
Adiu*n*ges num*er*u*m* que*m* p*re*bet duct*us* ear*um*.

leaf 159 *b*.

The fifth chapitre of the craft. ¶ Here is þe 5 case, þe quych is þis: yf *þe neþ*er* figur*e* schul m*u*ltiplie þe hier, and þat hier figur*e* is not recte ou*er* his hede. And þat neþ*er* figur*e* hase oþ*er* figures, or on figure ou*er* his hede by m*u*ltiplicacioɳ, þat hase be afor*e*, þou schalt write þat nounbre, þe quych comes of þat, ou*er* all*e* þe ylke figures hedes, as þus here: 236
234 Multiply 2 by 2, þat wol be 4; set 4 ou*er* þe hede of þat 2. þen [14] m*u*ltiplies þe hier 2 by þe neþ*er* 3, þat wol be 6. set ou*er* his hede 6, multiplie þe hier 2 by þe neþ*er* 4, þat wol be 8. do away þe hier 2, þe quych stondes ou*er* þe hede of þe figur*e* of 4, and set þ*er*e 8. And þou schalt haue þis nounb*re* here 46836
234 And antery þi figur*e*s, þat is to say, set þi neþ*er* 4 vnd*er* þe hier 3, and set þi 2 other figures ner*e* hym, so þat þe neþ*er* 2 stonde vnd*ur* þe hier 6, þe quych 6 stondes in þe lyft side. And þat 3 þat stondes vndur 8, as þus aftur 3e may se,
46836
234 Now worch forthermor*e*, And m*u*ltiplye þat hier 3 by 2,

þat wol be 6, set þ*a*t 6 þe quych stondes ou*er* þe hede of þat 2, And þen worch as I taȝt þe afore.

leaf 160 *a*.

*** ¶ Si sup*ra*posita cifra debet m*u*ltiplicar*e***
Prorsus ea*m* deles & ibi scribi cifra debet.

The sixth case of the craft. ¶ Here is þe 6. case, þe quych is þis: yf hit happe þat þe figur*e* by þe quych þou schal m*u*ltiplye þe hier figur*e*, þe quych stondes ryght ou*er* hym by a 0, þou schalt do away þat figur*e*, þe quych ou*er* þat cifre hede. ¶ And write þ*ere* þat nounbre þat comes of þe m*u*ltiplicacion̨ as þus, 23. do away 2 and sett þ*er*e a 0. vn*de* v*er*sus.

¶ Si cifra m*u*ltiplicat alia*m* posita*m* sup*er* ipsam
Sitq*ue* locus sup*ra* vacu*us* sup*er* hanc cifra*m* fiet.

The seventh case of the craft. ¶ Here is þe 7. case, þe quych is þis: yf a 0 schal m*u*ltiply a figur*e*, þe quych stondes not recte ou*er* hym, And ou*er* þat 0 stonde no thyng, þou schalt write ou*er* þat 0 anoþ*er* 0 as þus: 24

03 multiplye 2 be a 0, it wol be nothyng*e*. write þere a 0 ou*er* þe hede of þe neþ*er* 0, And þen worch forth til þou come to þe ende.

¶ Si sup*ra* [15] fuerit cifra sem*per* e*st* pr*ete*r*e*unda.

The eighth case of the craft. ¶ Here is þe 8. case, þe quych is þis: yf þ*ere* be a 0 or mony cifers in þe hier rewe, þ*o*u schalt not m*u*ltiplie hem, bot let hem stonde. And antery þe figures beneþe to þe next figur*e* sygnificatyf as þus: 00032.

22 Ou*er*-lepe all*e* þese cifers & sett þat leaf 160 *b*. *neþ*er* 2 þat stondes toward þe ryght side, and sett hym vnd*ur* þe 3, and sett þe oþ*er* nether 2 nere hym, so þat he stonde vnd*ur* þe thrydde 0, þe quych stondes next 3. And þan worch. vnd*e* v*er*sus.

¶ Si dubites, an sit b*ene* m*u*ltiplicac*io* facta,
Diuide totalem nu*mer*u*m* p*er* multiplicante*m*.

How to prove the multiplicacion.

¶ Here he teches how þou schalt know whe*þer* þou hase wel I-do or no. And he says þat þou schalt deuide all*e* þe nounb*r*e þat comes of þe m*u*ltiplicacioɳ by þe ne*þer* figures. And þen þou schalt haue þe same nounbur þat þ*o*u hadyst in þe begynnyng*e*. but ȝet þou hast not þe craft of dyuisioɳ, but þ*o*u schalt haue hit aft*er*warde.

¶ **P*er* num*er*u*m* si vis nu*mer*u*m* q*u*oq*ue* m*u*ltiplicar*e***
¶ **T*antu*m p*er* normas subtiles absq*ue* figuris**
Has normas pot*er*is p*er* v*er*sus scir*e* sequentes.

Mentall multiplicacion.

¶ Here he teches þe to m*u*ltiplie be þowȝt figures in þi mynde. And þe sentence of þis v*er*se is þis: yf þ*o*u wel m*u*ltiplie on nounbre by anoþ*er* in þi mynde, þ*o*u schal haue þ*er*eto rewles in þe v*er*ses þat schal come aft*er*.

¶ **Si tu p*er* digitu*m* digitu*m* vis m*u*ltiplicar*e***
Re*gula* p*re*cedens dat qualit*er* est op*er*andu*m*.

Digit by digit is easy.

¶ Here he teches a rewle as þou hast afor*e* to m*u*ltiplie a digit be anoþ*er*, as yf þou wolde wete qwat is sex tymes 6. þou leaf 161 *a.* *schalt wete by þe rewle þat I taȝt þe befor*e*, yf þou haue mynde þ*er*of.

¶ **Articulu*m* si p*er* reliquu*m* reliquu*m* vis m*u*lti*plica*r*e***
In p*r*o*p*riu*m* digitu*m* debet vt*er*q*ue* resolui.
¶ **Articul*us* digitos post se m*u*ltiplicantes**
Ex digit*us* quociens retenerit m*u*ltipli*cari*
Articuli faciu*n*t tot centu*m* m*u*ltiplicati.

The first case of the tenth.

¶ Here he teches þe furst rewle, þe quych is þis: yf þou wel m*u*ltiplie an articul be anoþ*er*, so þat both þe articuls bene w*ith*-Inne an hundreth, þus þ*o*u schalt do. take þe digit of

Articles by articles.

bothe the articuls, for eu*er*y articul hase a digit, þen m*u*ltiplye þat on digit by þat oþ*er*, and loke how mony vnytes ben in þe nounbre þat comes of þe m*u*ltiplicacioɳ of þe 2 digittes, & so mony hundrythes ben in þe nounb*re* þat schal come of þe m*u*ltiplicacioɳ of þe ylke 2 articuls as yf þ*o*u wold wete qwat is ten tymes ten. take þe

An example.

digit of ten, þe quych is 1; take þe digit of þat oþ*er* ten, þe quych is on. ¶ Also m*u*ltiplie 1 be 1, as on tyme on þat is but 1. In on is but on vnite as þou wost welle, þ*er*efor*e* ten tymes ten is but a hundryth. ¶ Also yf þou wold wete what is another example. twenty tymes 30. take þe digit of twenty, þat is 2; & take þe digitt of thrytty, þat is 3. m*u*ltiplie 3 be 2, þat is 6. Now in 6 ben 6 vnites, ¶ And so mony hundrythes ben in 20 tymes 30*, leaf 161 *b.* þ*ere*efor*e* 20 tymes 30 is 6 hundryth euen. loke & se. ¶ But yf it be so þat on*e* articul be w*ith*-Inne an hundryth, or by-twene an hundryth and a thowsande, so þat it be not a þowsande fully. þen loke how mony vnytes ben in þe nounbur þat comys of þe m*u*ltiplicacion. [16]And so mony tymes [16] of 2 digitt*es* of ylke articuls, so mony thowsant ben in þe nounbre, the qwych comes of þe m*u*ltiplicacion. And so mony tymes ten thowsand schal be in þe nounbre þat comes of þe m*u*ltiplicacion of 2 articuls, as yf þ*o*u wold wete qwat is 4 hundryth tymes [two hundryth]. Multiply 4 be 2, [17] þat wol be 8. in 8 ben 8 vnites. *How to work subtly without Figures.* Mental Explanation. ¶ And so many tymes ten thousand be in 4 hundryth tymes [2] [17] hundryth, þ*a*t is 80 thousand. Take hede, I schall telle þe a generalle rowle whan þ*o*u hast 2 another example. articuls, And þou wold wete qwat comes of þe m*u*ltiplicacion of hem 2. m*u*ltiplie þe digit of þ*a*t on articuls, and kepe þat nounbre, þen loke how mony cifers schuld go befor*e* þat on articuls, and he were write. Als mony cifers schuld go befor*e* þat other, & he were write of cifers. And haue all*e* þe ylke cifers toged*ur* in þi mynde, leaf 162 *a.* *a-rowe ychon aftur other, and in þe last plase set þe nounbre þat comes of þe m*u*ltiplicacion of þe 2 digittes. And loke in þi mynde in what place he stondes, where in þe secunde, or in þe thryd, or in þe 4, or where ellis, and loke qwat þe figures by-token in þat place; & so mych is þe nounbre þat another example. comes of he 2 articuls y-m*u*ltiplied to-ged*ur* as þus: yf þ*o*u wold wete what is 20 thousant tymes 3 þowsande. m*u*ltiply þe digit of þat articull*e* þe quych is 2 by þe digitte of þat oþ*er* articul þe quych is 3, þat wol be 6. þen loke how mony

cifers schal go to 20 thousant as hit schuld be write in a tabul. c*er*tainly 4 cifers schuld go to 20 þowsant. ffor þis figure 2 in þe fyrst place betokenes twene. ¶ In þe secunde place

Notation.

hit betokenes twenty. ¶ In þe 3. place hit betokenes 2 hundryth. .¶. In þe 4 place 2 thousant. ¶ In þe 5 place h*i*t betokenes twenty þousant. þ*ere*for*e* he most haue 4 cifers a-for*e* hym þat he may sto*n*de in þe 5 place. kepe þese 4 cifers in thy mynde, þen loke how mony cifers goꝛ to 3 thousant. Certayn to 3 thousante leaf 162 *b.* *goꝛ 3 cifers afor*e*. Now cast ylke 4 cifers þat schuld go to twenty thousant, And thes 3 cifers þat schuld go afor*e* 3 thousant, & sette hem in rewe ychoꝛ aft*er* oþ*er* in þi mynde, as þai schuld stonde in a tabull*e*. And þen schal þou haue 7 cifers; þen sett þat 6 þe quych comes of þe m*u*ltiplicacioꝛ of þe 2 digitt*es* aft*u*r þe ylke cifers in þe 8 place as yf þat hit stode in a tabul. And loke qwat a figur*e* of 6 schuld betoken in þe 8 place. yf hit wer*e* in a tabul & so mych it is. & yf þat figure of 6 stonde in þe fyrst place he schuld betoken but 6. ¶ In þe 2 place he schuld betoken sexty. ¶ In the 3 place he schuld betokeꝛ sex hundryth. ¶ In þe 4 place sex thousant. ¶ In þe 5 place sexty

Notation again.

þowsant. ¶ In þe sext place sex hundryth þowsant. ¶ In þe 7 place sex þowsant thousant*es*. ¶ In þe 8 place sexty þowsant thousantes. þ*er*for*e* sett 6 in octauo loco, And he schal betoken sexty þowsant thousantes. And so mych is

Mental multiplication.

twenty þowsant tymes 3 thousant, ¶ And þis rewle is gen*er*all*e* for all*e* man*er* of articuls, Whethir þai be hundryth or þowsant; but þ*o*u most know well þe craft of þe wryrchyng*e* in þe tabull*e* leaf 163 *a.* *or þou know to do þus in þi mynde aftur þis rewle. Thou most þat þis rewle holdyþe note but wher*e* þ*ere* ben 2 articuls and no mo of þe quych ayther of hem hase but on figur*e* significatyf. As twenty tymes 3 thousant or 3 hundryth, and such oþ*ur*.

¶ Articulum digito si m*u*ltiplicare o*portet*
Articuli digit[i sumi quo multiplicate]
Debem*us* reliquu*m* quod m*u*ltiplicat*ur* ab ill*is*

P*er* reliq*u*o decuplu*m* sic su*m*ma*m* later*e* neq*ui*bi*t*.

The third case of the craft.

¶ Here he puttes þe thryde rewle, þe quych is þis. yf þ*o*u wel m*u*ltiply in þi mynde, And þe Articul be a digitte, þou schalt loke þat þe digitt be w*ith*-Inne an hundryth, þen þou schalt m*u*ltiply the digitt of þe Articulle by þe oþer digitte. And eu*er*y vnite in þe nounbre þat schall*e* come þ*ere*-of schal betoken ten. As þus: yf þat þou wold wete qwat is twyes 40. m*u*ltiplie þe digitt*e* of 40, þe quych is 4, by þe oþ*er* diget, þe quych is 2. And þat wolle be 8. And in þe nombre of 8 ben 8 vnites, & eu*er*y of þe ylke vnites schuld stonde for 10. þ*ere*-fore þ*ere* schal be 8 tymes 10, þat wol be 4 score. And so mony is twyes 40. ¶ If þe articul be a hundryth or be 2 hundryth And a þowsant, so þat hit be notte a thousant, leaf 163 *b*. *worch as þo*u* dyddyst afor*e*, saue þ*o*u schalt rekene eu*er*y vnite for a hundryth.

An example.

¶ In nu*mer*u*m* mixtu*m* digitu*m* si ducer*e* cures
Articul*us* mixti sumat*ur* deinde resoluas
In digitu*m* post fac respectu de digitis
Articul*usque* docet excrescens in diriua*n*do
In digitu*m* mixti post ducas m*u*ltiplica*n*te*m*
¶ De digitis vt norma [18][docet] de [hunc]
Multiplica si*mu*l et sic postea summa patebit.

The fourth case of the craft.

Here he puttes þe 4 rewle, þe quych is þis: yf þou m*u*ltipliy on composit be a digit as 6 tymes 24, [19]þen take þe diget of þat composit, & m*u*ltiply þ*a*t digitt by þat oþ*er* diget, and kepe þe nomb*ur* þat comes þ*ere*-of. þen take þe digit of þat composit, & m*u*ltiply þat digit by anoþ*er* diget, by þe quych þ*o*u hast m*u*ltiplyed þe diget of þe articul, and loke qwat comes þ*ere*-of. þen take þou þat nounbur, & cast hit to þat other nounbur þat þ*o*u secheste as þus yf þou wel wete qwat comes of 6 tymes 4 & twenty. multiply þat articull*e* of þe composit by þe digit, þe quych is 6, as yn þe thryd rewle þou was tauȝt, And þat schal be 6 scor*e*. þen m*u*ltiply þe diget of þe *com*posit, leaf 164 *a*. *þe quych is 4,

Composite by digit.

Mental multiplication.

and m*u*ltiply þat by þat other diget, þe quych is 6, as þou wast tauȝt in þe first rewle, yf þ*o*u haue mynde þ*er*of, & þat wol be 4 & twenty. cast all ylke nounburs to-ged*ir*, & hit schal be 144. And so mych is 6 tymes 4 & twenty.

to multiply without Figures.

¶ Duct*us* in articulu*m* num*erus* si *com*posit*us* sit
Articulu*m* puru*m* comites articulu*m* q*uoque*
Mixti pro digit*is* post fiat [et articulus vt]
Norma iubet [retinendo quod extra dicta ab illis]
Articuli digitu*m* post tu mixtu*m* digitu*m* duc
Re*gula* de digitis nec p*re*cipit articul*us*q*ue*
Ex quib*us* exc*re*scens su*m*me tu iunge p*ri*ori
Sic ma*n*ifesta cito fiet t*ibi* su*m*ma petita.

The fifth case of the craft.

¶ Here he puttes þe 5 rewle, þe quych is þis: yf þ*o*u wel m*u*ltiply an Articul be a composit, m*u*ltiplie þat Articul by þe articul of þe composit, and worch as þou wos tauȝt in þe secunde rewle, of þe quych rewle þe v*er*se begynnes þus.

Article by Composite.

¶ Articulu*m* si p*er* Relicu*m* vis m*u*ltiplicare. þen m*u*ltiply þe diget of þe composit by þat oþ*ir* articul aft*ir* þe doctrine of þe 3 rewle. take þ*er*of gode hede, I p*ra*y þe as þus. Yf þ*o*u wel wete what is 24 tymes ten. Multiplie ten by 20, þat wel be 2 hundryth. þen m*u*ltiply þe diget of þe 10, þe quych is 1, by þe diget of þe composit, þe quych is 4, & þ*a*t leaf 164 *b*. *wol be 4. þen reken eu*er*y vnite þat is in 4 for 10, & þat schal be 40. Cast 40 to 2 hundryth, & þat wol be 2 hundryth & 40. And so mych is 24 tymes ten.

An example.

to work without Figures.

¶ Compositu*m* num*erum* mixto si[c] m*u*ltiplicabis
Vndecies tredeci*m* sic e*st* ex hiis op*er*andum
In reliquu*m* p*rimu*m demu*m* duc post in eund*em*
Vnu*m* post den*u*m duc in t*ri*a dei*n*de p*er* vnu*m*
Multiplices*que* dem*u*m int*ra* o*mn*ia m*u*ltiplicata
In su*m*ma decies q*ua*m si fu*er*it t*ibi* doces

Multiplicandor*um* de normis sufficiunt h*ec*.

The sixth case of the craft. ¶ Here he puttes þe 6 rewle, & þe last of all*e* multiplicacioɳ, þe quych is þis: yf þ*o*u wel m*u*ltiplye a *com*posit by a-noþ*er* composit, þou schalt do þus. Composite multiplie þat on composit, qwych þ*o*u welt of the twene, by þe articul of þe toþ*er* composit, as þ*o*u were tauȝt in þe 5 rewle, þen m*u*ltiplie þ*a*t same composit, þe quych þou hast m*u*ltiplied by þe oþ*er* articul, by þe digit of þe oþ*er* composit, as þou was tauȝt in þe 4 rewle. Exemplum. Now, yf þou wold wete what is 11 tymes 13, as þ*o*u was tauȝt in þe 5 rewle, & þat schal be an hundryth & ten, aft*er*warde m*u*ltiply þat same co*m*posit þ*a*t þ*o*u hast m*u*ltiplied, þe quych is a .11. And m*u*ltiplye hit be þe digit of þe oþ*er* composit, þe quych is 3, for 3 is þe digit of 13, And þat wel be 30. þen take þe digit of þat composit, þe quych composit þou m*u*ltiplied by þe digit of þ*a*t oþ*er* *com*posit, leaf 165 *a.* *þe quych is a 11. ¶ Also of the quych 11 on is þe digit. m*u*ltiplie þat digitt by þe digett of þat oth*er* composit, þe quych diget is 3, as þ*o*u was tauȝt in þe first rewle i*n* þe begynnyng*e* of þis craft. þe quych rewle begynn*es* "In digitu*m* cures." And of all*e* þe m*u*ltiplicacioɳ of þe 2 digitt comys thre, for onys 3 is but 3. Now cast all*e* þese nounbers toged*ur*, the quych is þis, a hundryth & ten & 30 & 3. And al þat wel be 143. Write 3 first in þe ryght side. And cast 10 to 30, þat wol be 40. set 40 next aft*ur* towarde þe lyft side, And set aftur a hundryth as her*e* an Ensampull*e*, 143.

(Cetera desunt.)

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1. In MS, 'awiy.'
2. 'ben' repeated in MS.
3. In MS. 'thausandes.'
4. Perhaps "So."

5. ‘hali’ marked for erasure in MS.

6. ‘moy’ in MS.

7. ‘Subt*ra*has a*u*t addis a dext*ri*s *ve*l medi*a*b*is*’ added on margin of MS.

8. After ‘craft’ insert ‘the .4. what is þe p*ro*fet of þis craft.’

9. After ‘sythes’ insert ‘& þis wordes fyue sithe & sex sythes.’

10. ‘t’l’ marked for erasure before ‘tyl’ in MS.

11. Here ‘of þe same rew’ is marked for erasure in MS.

12. ‘s*ed*’ deleted in MS.

13. 6883 in MS.

14. ‘þen’ overwritten on ‘þat’ marked for erasure.

15. ‘Supra’ inserted in MS. in place of ‘cifra’ marked for erasure.

16–16. Marked for erasure in MS.

17. 4 in MS.

18. docet. decet MS.

19. ‘4 times 4’ in MS.

The Art of Nombryng.

A TRANSLATION OF

John of Holywood's De Arte Numerandi.

[*Ashmole MS. 396, fol. 48.*]

Boys seying in the begynnyng of his Arsemetrik*e*:—

All*e* Fol. 48. thynges that ben*e* fro the first begynnyng of thynges have p*ro*cedede, and come forth*e*, And by reso*u*n of nombre ben formed*e*; And in wise as they ben*e*, So oweth*e* they to be knowen*e*; wherfor in vniu*er*sall*e* knowlechyng of thynges the Art of nombrynge is best, and most operatyf*e*.

Therfore sithen the science of the whiche at this tyme we intendene to write of standith*e* all*e* and about nombre:

The name of the art.

ffirst we most se, what is the p*ro*pre name therof*e*, and fro whens the name come: Afterward*e* what is nombre, And how manye spices of nombre ther ben. The name is cleped*e* Algorisme,

Derivation of Algorism.

hade out of Algor*e*, other of Algos, in grewe, That is clepid*e* in englissh*e* art other craft, And of Rithm*us* that is called*e* nombre. So algorisme is cleped*e* the art of nombryng,

Another.

other it is had of*e* en or in, and gogos that is introduccio*u*n, and Rithm*us* nombre, that is to say Interduccio*u*n of nombre.

Another.

And thirdly it is had*e* of the name of a kyng that is cleped*e* Algo and Rythm*us*; So called*e* Algorism*us*.

Kinds of numbers.

Sothely .2. maner*e* of nombres ben notified*e*; Formall*e*, [1] as nombr*e* i*s* vnitees gadred*e* to-gedres; Materiall*e*, [2] as nombr*e* is a collecci*ou*n of vnitees. Other

nombr*e* is a multitude had*e* out of vnitees, vnitee is that thynge wher-by eu*er*y thynge is called*e* oone, other o thynge. Of nombres, that one is cleped*e* digitall*e*, that other*e* Article, Another a nombre componed*e* oþ*er* myxt. Another digitall*e* is a nombre w*ith*-in .10.; Article is þ*a*t nombre that may be dyvyded*e* in .10. p*ar*ties egally, And that there leve no residue; Componed*e* or medled*e* is that nombre that is come of a digite and of an article. And vndrestand*e* wele that all*e* nombres betwix .2. articles next is a nombr*e* componed*e*. The 9 rules of the Art. Of this art ben*e* .9. spices, that is forto sey, num*er*acio*u*n, addicio*u*n, Subtraccio*u*n, Mediac*io*u*n, Duplacio*u*n, Multipliacio*u*n, Dyvysio*u*n, Progressio*u*n, And of Rootes the extraccio*u*n, and that may be had*e* in .2. maners, that is to sey in nombres quadrat, and in cubic*es*: Amonge the which*e*, ffirst of Num*er*acio*u*n, and aft*er*ward*e* of þe oþ*er*s by ordure, y entende to write.

ter I. Numeration.

Fol. 48 *b*.

***For-soth*e* num*er*acio*u*n is of eu*er*y numbre by competent figures an artificiall*e* rep*re*sentacio*u*n.**

Figures, differences, places, and limits. Sothly figure, difference, places, and lynes supposen o thyng other the same, But they ben sette here for dyue*r*s resons. ffigure is cleped*e* for p*ro*traccio*u*n of figuracio*u*n; Difference is called*e* for therby is shewed*e* eu*er*y figure, how it hath*e* difference fro the figures before them: place by cause of space, where-in me writeth*e*: lynees, for that is ordeyned*e* for the p*re*sentacio*u*n of eu*er*y figure. The figures. And vnderstonde that ther ben .9. lymytes of figures that rep*re*senten the .9. digit*es* that ben these. 0. 9. 8. 7. 6. 5. 4. 3. 2. 1. The cipher. The .10. is cleped*e* theta, or a cercle, other a cifre, other a figure of nought for nought it signyfieth*e*. Nathelesse she holdyng that place giveth*e* others for to signyfie; for with*e*-out cifre or cifres a pure article may not be writte. The

Numeration.

And sithen that by these .9. figures significatif*es* Ioyned*e* w*ith* cifre or w*ith* cifres all*e* nombres ben and may be rep*re*sented*e*, It was, nether is, no nede to fynde any more figures or signes.

Of digits.

And note wele that eu*er*y digite shall*e* be writte w*ith* oo figure allone to it ap*ro*pred.

Of articles.

And all*e* articles by a cifre, ffor eu*er*y article is named*e* for oone of the digitis as .10. of 1.. 20. of. 2. and so of the others, &c. And all*e* nombres digitall*e* owen to be sette in the first difference: All*e* articles in the seconde. Also all*e* nombres fro .10. til an .100. [which] is excluded*e*, with .2. figures mvst be writte; And yf it be an article, by a cifre first put, and the figure y-writte toward*e* the lift hond*e*, that signifieth*e* the digit of the which*e* the article is named*e*.

Of compositas.

And yf it be a nombre componed*e*, ffirst write the digit that is a part of that componed*e*, and write to the lift side the article as it is seid*e* be-fore. All*e* nombre that is fro an hundred*e* tille a thousand*e* exclused*e*, owith*e* to be writ by .3. figures; and all*e* nombre that is fro a thousand*e* til .x. Mł. mvst be writ by .4. figures; And so forthe.

The value of position.

And vnderstond*e* wele that eu*er*y figure sette in the first place signyfieth*e* his digit; In the second*e* place .10. tymes his digit; In the .3. place an hundred*e* so moche; In the .4. place a thousand*e* so moche; In the .5. place .x. thousand*e* so moch*e*; In the .6. place an hundred*e* thousand*e* so moch*e*; In the .7. place a thousand*e* thousand*e*. And so infynytly mvltiplying by *these .3. 10, 100, 1000. And vnderstand*e* wele that competently me may sette vpon figure in the place of a thousand*e*, a prik*e* to shewe how many thousand*e* the last figure shall*e* rep*re*sent.

Fol. 49.

Numbers are written from right to left.

We writen*e* in this art to the lift side-ward*e*, as arabien*e* writen*e*, that weren fynders of this science, other*e* for this reso*u*n, that for to kepe a custumable ordr*e* in redyng, Sette we all*e*-wey the more nombre before.

ter II. Addition.

Addicioun is of nombre other of nombres vnto nombre or to nombres aggregacio*u*n, that me may see that that is come therof as exc*re*ssent. In addicio*u*n, 2. ordres of figures and .2. nombres ben necessary, that is to sey, a nombre to be added*e* and the nombre wherto the addic*i*oun sholde be made to. The nombre to be added*e* is that þat sholde be added*e* therto, and shall*e* be vnderwriten; the nombre vnto the which*e* addicio*u*n shall*e* be made to is that nombre that resceyueth*e* the addicion of þat other, and shall*e* be writen above; and it is convenient that the lesse nombre be vnderwrit, and the more added*e*, than the contrary. But whether it happ*e* one other other, the same comyth*e* of, Therfor, yf þow wilt adde nombre to nombre, write the nombre wherto the addicio*u*n shall*e* be made in the omest ordre by his differences, so that the first of the lower ordre be vndre the first of the omyst ordre, and so of others. That done, adde the first of the lower ordre to the first of the omyst ordre. And of such*e* addicio*u*n, other þ*er*e grow*ith* therof a digit, An article, other a composed*e*. If it be digitus, In the place of the omyst shalt thow write the digit excrescyng, as thus:—

Definicion

How the numbers should be written.

The manner of working.

Begin adding right.

The resultant	2
To whom it shal be added*e*	1
The nombre to be added*e*	1

If the article, in the place of the omyst put a-way by a cifre writte, and the digit transferred*e*, of þe which*e* the article toke his name, toward*e* the lift side, and be it added*e* to the next figure folowyng, yf ther be any figure folowyng; or no, and yf it be not, leve it [in the] void*e*, as thus:—

The manner of working,

or an article,

The resultant	10
To whom it shall*e* be added*e*	7

The nombre to be added*e*	3

Resultans	2	7	8	2	7
Cui d*ebet* addi	1	0	0	8	4
Num*erus* addend*us*	1	7	7	4	3

And yf it happe that the figure folowyng wherto the addicio*u*n shall*e* be made by [the cifre of] an article, it sette a-side;

The resultant	17
To whom it shall*e* be added*e*	10
The nombre to be added*e*	7

In his place write the Fol. 49 *b.* *[digit of the] Article as thus: —

And yf it happe that a figure of .9. by the figure that me mvst adde [one] to,

The resultant	10
To whom it shall*e* be added*e*	9
The nombre to be added*e*	1

In the place of that 9. put a cifre *and* write þe article toward*e* þe lift hond*e* as bifore, and thus:—

And yf [3] dry compositt. [there from grow a] nombre componed, [4] [in the place of the nombre] put a-way [5]

The resultant	12
To whom it shall*e* be added*e*	8
The nombre to be added*e*	4

[let] the digit [be] [6] writ þ*a*t is part of þ*a*t co*m*posid*e*, and þan put to þe lift side the article as before, and þus:—

The translatour, adde.

This done, adde the seconde to the second*e*, and write above o þ*er* as before. Note wele þ*a*t in addic*i*ons and in all*e*

spices folowyng, whan he seith*e* one the other shall*e* be writen aboue, and me most vse eu*er* figure, as that eu*er*y figure were sette by half*e*, and by hym-self*e*.

ter III. Subtraction.

Definition of Subtraction.

Subtracciou*n* is of .2. p*ro*posed*e* nombres, the fyndyng of the excesse of the more to the lasse: Other subtracciou*n* is ablaciou*n* of o nombre fro a-nother, that me may see a some left. The lasse of the more, or even of even, may be w*ith*draw; The more fro the lesse may neu*er* be.

How it may be done.

And sothly that nombre is more that hath*e* more figures, So that the last be signyficatife*s*: And yf ther ben as many in that one as in that other, me most deme it by the last, other by the next last.

What is required.

More-ou*er* in w*ith*-drawyng .2. nombres ben necessary; A nombre to be w*ith*draw, And a nombre that me shall*e* w*ith*-draw of. The nombre to be w*ith*-draw shall*e* be writ in the lower ordre by his differences;

Write the greater number above.

The nombre fro the which*e* me shall*e* with*e*-draw in the omyst ordre, so that the first be vnder the first, the second*e* vnder the second*e*, And so of all*e* others.

Subtract the first figure if possible.

Withe-draw therfor the first of the lower*e* ordre fro the first of the ordre above his hede, and that wolle be other more or lesse, oþ*er* egall*e*.

The remanent	20
Wherof me shall*e* w*ith*draw	22
The nombre to be w*ith*draw	2

The remanent	2	2
Wherof me shall*e* w*ith*-draw	2	8
Þe nombre to be w*ith*draw		6

yf it be egall*e* or even the figure sette beside, put in his place a cifre. And yf it be more put away þ*er*fro als many of

vnitees the lower figure conteynethe, and writ the residue as thus

Fol. 50.

*Remanen*ns*	2	2	1	8	2	9	9	9	8
A quo sit subtraccio	8	7	2	4	3	0	0	0	4
Numerus subt*ra*hend*us*	6	5	7	[6]	.	.	.	.	6

If it is not possible to borrow ten,

And yf it be lesse, by-cause the more may not be w*it*h-draw ther-fro, borow an vnyte of the next figure that is worth*e* 10. Of that .10. and of the figure that ye wold*e* have w*it*h-draw fro be-fore to-gedre Ioynede*e*,

and then subtract.

The remanent	1	8
Wherof me shall*e* w*ith*-draw	2	4
The nombre to be w*ith*-draw	0	6

w*ith*-draw þe figure be-nethe, and put the residue in the place of the figure put a-side as þ*us*:—

If the second figure is one

And yf the figure wherof me shal borow the vnyte be one, put it a-side, and write a cifre in the place þ*er*of, lest the figures folowing faile of thair*e* nombre, and þan worch*e* as it shew*ith* in this figure here:—

The remanent	3	0	9[8]
Wherof me shal w*ith*-draw	3	1	2
The nombre to be w*ith*-draw	.	.	3

If the second figure is a cipher

And yf the vnyte wherof me shal borow be a cifre, go ferther to the figure signyficatif*e*, and ther borow one, and reto*ur*nyng bak*e*, in the place of eu*er*y cifre þ*a*t ye passid*e* ou*er*, sette figures of .9. as here it is specified*e*:—

The remenaunt	2	9	9	9	9
Wherof me shall*e* w*ith*-draw	3	0	0	0	3
The nombre to be w*ith*-draw					4

And whan me comethe to the nombre wherof me intendithe, there remaynethe alle-wayes .10. ffor þe whiche .10. &c.

A justification of the reason given. The reson why þat for euery cifre left behynde me setteth figures ther of .9. this it is:—If fro the .3. place me borowede an vnyte, that vnyte by respect of the figure that he came fro representith an .C., In the place of that cifre [passed over] is left .9., [which is worth ninety], and yit it remaynethe as .10., And the same resone wolde be yf me hade borowede an vnyte fro the .4., .5., .6., place, or ony other so vpwarde. This done, withdraw the seconde of the lower ordre fro the figure above his hede of þe omyst ordre, and wirche as before.

Why it is better to work from right to left. And note wele that in addicion or in subtraccioun me may wele fro the lift side begynne and ryn to the right side, But it wol be more profitabler to be do, as it is taught.

How to prove your work. And yf thow wilt prove yf thow have do wele or no, The figures that thow hast withdraw, adde them ayene to the omyst figures, and they wolle accorde with the first that thow haddest yf thow have labored wele, and in like wise in addicioun, whan thow hast addede alle thy figures, withdraw them that thow first Fol. 50 *b.* *addest, and the same wolle retourne. The subtraccioun is none other but a prouffe of the addicioun, and the contrarye in like wise.

ter IV. Mediation.

Definition of mediation. Mediacioun is the fyndyng of the halfyng of euery nombre, that it may be seyne what and how moche is euery halfe. In halfyng ay oo order of figures and oo nombre is necessary, that is to sey the nombre to be halfede. Therfor yf thow wilt half any nombre, write that nombre by his differences, and begynne at the right, that is to sey, fro the first figure to the right side, so that it be signyficatife other represent vnyte or eny other digitalle nombre. If it be vnyte write in his place a cifre for the figure folowyng, [lest

Where to begin.

If the figure be odd or even.

they signify less], and write that vnyte w*ith*out in the table, other resolue it in .60. mynvt*es* and sette a-side half of tho m*inutes* so, and reserve the remen*au*nt w*ith*out in the table, as thus .30.; other sette w*ith*out thus .*dī*: that kepeth*e* none ordre of place, Nathelesse it hath*e* signyficacio*u*n. And yf the other figure signyfie any other digital nombre fro vnyte forth*e*, oþ*er* the nombre is od*e* or even*e*.

Halfed*e*	2	2
to be halfed*e*	4	4

halfed*e*	2	3	[di]
To be halfed*e*	4	7	

What to do if it be even, write this half in this wise:—

If it be even,

And if it be odde, Take the next even vndre hym conteyned*e*, and put his half in the place of that odde, and of þe vnyte that remayneth*e* to be halfed*e* do thus:—

Then halve this second figure.

This done, the second*e* is to be halfed*e*, yf it be a cifre put it be-side, and yf it be significatif*e*, other it is even or od*e*: If it be even, write in the place of þe nombres wiped*e* out the half*e*; yf it be od*e*, take the next even vnder it co*n*tenyth*e*, and in the place of the Impar sette a-side put half of the even: The vnyte that remayneth*e* to be halfed*e*, respect had*e* to them before, is worth*e* .10.

Halfed*e*			
to be halfed*e*			

If it is odd, add 5 to the figure before it.

Dyvide that .10. in .2., 5. is, and sette a-side that one, and adde that other to the next figure p*re*cedent as here:—

And yf þe addicio*u*n sold*e* be made to a cifre, sette it a-side, and write in his place .5.

doubled*e*	2	6	8	9	0	10	17	4

to be doublede	1	3	4	4	5	5	8	7

And vnder this fo*ur*me me shall*e* write and worch*e*, till*e* the totall*e* nombre be halfed*e*.

ter V. Duplation.

Definicioun of Duplacioun.

Duplicacio*un* is ag*re*gacion of nombre [to itself] þat me may se the nombre growen. In doublyng*e* ay is but one ordre of figures necessarie. And me most be-gynne w*ith* the lift side, other of the more figure, And after the nombre of the more figure rep*re*sentith*e*. Fol. 51. *In the other .3. before we begynne all*e* way fro the right side and fro the lasse nombre,In this spiece and in all*e* other folowyng we wolle begynne fro the lift side, ffor and me bigon th*e* double fro the first, omwhile me myght double oo thyng and two wyle that me myght double fro the right, that wold*e* be harder in techyng and in workyng. Therfor yf thow wolt double any nombre, write that nombre by his differences, and double the last. And of that doubly*n*g other growith*e* a nombre digital, article, or componed*e*. [If it be a digit, write it in the place of the first digit.]

Where to begin.

And how it is.

double	10
to be doublede	5

What to do with an article.

If it be article, write in his place a cifre and transferre the article toward*e* the lift, as thus:—

And yf the nombre be componed*e*,

doublede	16
to be doublede	8

write a digital that is part of his composicio*u*n, and sette the article to the lift hand*e*, as thus:—

That done, me most double the last save one, and what growethe þerof me most worche as before. And yf a cifre be, touche it not. But yf any nombre shalle be addede to the cifre,

doublede	6	0	6
to be doublede	3	0	3

in þe place of þe figure wipede out me most write the nombre to be addede, as thus:—

In the same wise me shalle worche of alle others. How to
And this probacioun: prove your

Doublede	6	1	8
to be doublede	3	0	9

If thow truly double the halfis, and truly half the doubles, the same nombre and figure shalle mete, suche as thow laboured vpone first, And of the contrarie.

ter VI. Multiplication.

Definition of Multiplicacion. Multiplicacioun of nombre by hym-self other by a-nother, with proposide .2. nombres, [is] the fyndyng of the thirde, That so oft conteynethe that other, as ther ben vnytes in the oþer. In multiplicacioun .2. nombres pryncipally ben necessary, that is to sey, the nombre multiplying and the nombre to be multiplied, as here;—twies fyve. The nombre multiplying] is designed aduerbially The nombre to be multiplied resceyvethe a nominalle appellacioun, as twies .5. 5. is the nombre multiplied, and twies is the nombre to be multipliede.

Resultans	9	1	0	1	3	2	6	6	8	0	0	8
Multiplicandus	.	.	5	.	.	4	.	3	4	0	0	4
Multiplicans	.	2	2	.	3	3	2	2	2	.	.	.

Also me may thervpon*e* to assigne the. 3. nombre, the which*e* is Product Fol. 51 *b.* *clepe*d*e* p*ro*duct or p*ro*venient, of takyng out of one fro another: as twyes .5 is .10., 5. the nombre to be multiplie*d*e, and .2. the multipliant, and. 10. as before is come therof. And vnderstonde wele, that of the multipliant may be made the nombre to be multiplie*d*e, and of the contrarie, remaynyng eu*er* the same some, and herof*e* cometh*e* the comen speche, that seith*e* all nombre is convert*e*d*e* by Multiplying in hym self.

The Table of Multiplication.

There are 6 rules of Multiplication.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	10[10]	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	56	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

And ther ben .6 rules of Multiplicacio*u*n; ffirst, yf a digit multiplie a digit, considr*e* how many of vnytees ben betwix the digit by multiplying and his .10. beth*e* to-gedre accompted*e*, and so oft w*ith*-draw the digit multiplying, vnder the article of his deno*min*acio*u*n. Example of grace. If thow wolt wete how moch*e* is .4. tymes .8., [11]se how many vnytees ben betwix .8. [12] and .10. to-geder rekened*e*, and it shew*ith* that .2.: withdraw ther-for the quat*er*nary, of the article of his deno*min*acion twies, of .40., And ther remayneth*e* .32., that is, to some of all*e* the multiplicacio*u*n.

Where-upon for more evidence and declaracion the seid*e* table is made. When a digit multiplieth*e* an article, thow most bryng the digit into þe digit, of þe which*e* the article [has] [13] his name, and eu*er*y vnyte shall*e* stond*e* for .10., and

eu*er*y article an .100. (3) Compositely digit. When the digit multiplieth*e* a nombre componed*e*, þ*o*u most bryng the digit into ai þ*er* part of the nombre componed*e*, so þ*a*t digit be had into digit by the first rule, into an article by þe second*e* rule; and aft*er*ward*e* Ioyne the p*ro*duccio*u*n, and þ*er*e wol be the some totall*e*.

Resultans	1	2	6	7	3	6	1	2	0	1	2	0	8
Multiplicand*us*			2		3	2			6				4
Multiplicans		6	3	2	3			2	0		3	0	2

(4) Vnite by article. When any article multiplieth*e* an article, the digit wherof he is named*e* is to be brought Into the digit wherof the oþ*er* is named*e*, and eu*er*y vnyte wol be worth*e* Fol. 52. *an .100., and eu*er*y article. a .1000. (5) Compositely article. When any article multiplieth*e* a nombre componed*e*, thow most bryng the digit of the article into aither part of the nombre componed*e*; and Ioyne the p*ro*duccio*u*n, and eu*er*y article wol be worth*e* .100., and eu*er*y vnyte .10., and so woll*e* the some be open*e*.

(6) Compositely compositely. When a nombre componed*e* multiplieth*e* a nombre componed*e*, eu*er*y p*ar*t of the nombre multiplying is to be had*e* into eu*er*y p*ar*t of the nombre to be multiplied*e*, and so shall*e* the digit be had*e* twies, onys in the digit, that other in the article. The article also twies, ones in the digit, that other in the article. Therfor yf thow wilt any nombre by hym-self other by any other multiplie, write the nombre to be multiplied*e* in the ou*er* ordre by his differences, How to set down yor numbers. The nombre multiplying in the lower ordre by his differences, so that the first of the lower ordre be vnder the last of the ou*er* ordre. This done, of the multiplying, the last is to be had*e* into the last of the nombre to be multiplied*e*. Wherof than wolle grow a digit, an article, other a nombre componed*e*. If the result is a digit,

The resultant	6
To be multiplied*e*	3

Þe nombre multipliyng	2

If it be a digit, even above the figure multiplying is hede write his digit that come of, as it apperethe here:—

And yf an article had be writ ouer the figure multiplying his hede, put a cifre þer and transferre the article towarde the lift hande, as thus:—

The resultant	1	0
to be multiplied*e*		5
þe nombre m*u*ltipliyng		2

And yf a nombre componede be writ ouer the figure multyplying is hede, write the digit in the nombre componede is place, and sette the article to the lift hande, as thus:—

Resultant	1	2
to be multiplied*e*		4
the nombre multipliyng		3

Multiply next by the last but one, and so forth. This done, me most bryng the last save one of the multipliyng into the last of þe nombre to be multiplied*e*, and se what comythe therof as before, and so do w*ith* alle, tille me come to the first of the nombre multiplying, that must be brought into the last of the nombre to be multiplied*e*, wherof growithe oþer a digit, an article, Fol. 52 *b.* *other a nombre componede.

Resultant	6	6
to be multiplied*e*		3
the nombre m*u*ltipliyng	2	2

The resultant	1	1	0
to be multiplied*e*			5
þe nombre m*u*ltiplying		2	2

The resultant	1	3[15]	2
to be m*u*ltiplied*e*			4
þe nombr*e* m*u*ltiplia*n*t		3	3

If it be a digit, In the place of the ou*er*er, sette a-side, as here:

If an article happe, there put a cifre in his place, and put hym to the lift hand*e*, as here:

If it be a nombre componed*e*, in the place of the ou*er*er sette a-side, write a digit that[14] is a p*ar*t of the componed*e*, and sette on the left hond*e* the article, as here:

Then antery the multiplicatione, &c.

That done, sette forward*e* the figures of the nombre multiplying by oo difference, so that the first of the multipliant be vnder the last save one of the nombre to be multiplied*e*, the other by o place sette forward*e*. Than me shall*e* bryng*e* the last of the m*u*ltipliant in hym to be multiplied*e*, vnder the which*e* is the first multipliant. Work as before.

And than wolle growe oþ*er* a digit, an article, or a componed*e* nombre. If it be a digit, adde hym even above his hede; If it be an article, transferre hym to the lift side; And if it be a nombre componed*e*, adde a digit to the figure above his hede, and sette to the lift hand*e* the article. And all*e*-wayes eu*er*y figure of the nombre multipliant is to be brought to the last save one nombre to be multiplied*e*, til me come to the first of the multipliant, where me shall*e* wirche as it is seid*e* before of the first, and aft*er*ward*e* to put forward*e* the figures by o difference and one till*e* they all*e* be multiplied*e*. How to deal with ciphers.

And yf it happe that the first figure of þe multipliant be a cifre, and boue it is sette the figure signyficatif*e*, write a cifre in the place of the figur*e* sette a-side, as thus, *et*c.:

The resultant	1	2	0
to be multiplied*e*			6
the multipliant		2	0

How to deal with ciphres.

And yf a cifre happe in the lower order be-twix the first and the last, and even above be sette the fig*ur*e signyficatif,

The resultant	2	2	6	4	4
To be multiplied*e*			2	2	2
The multipliant	1	0	2		

leve it vntouched*e*, as here:—

And yf the space above sette be void*e*, in that place write thow a cifre. And yf the cifre happe betwix þe first and the last to be m*u*ltiplied*e*, me most sette forward*e* the ordre of the figures by thair*e* differences, for oft of duccio*u*n of figur*e*s in cifres nought is the resultant, as here,

Resultant	8	0	0	8	
to be m*u*ltiplied*e*	4	0	0	4	
the m*u*ltipliant	2	.	.	.	

Fol. 53. *wherof it is evident and open, yf that the first figure of the nombre be to be multiplied*e* be a cifre, vndir it shall*e* be none sette as here:—

Resultant	3	2	0^{16}
To be m*u*ltiplied*e*		8	0
The m*u*ltipliant		4	

Leave room between the rootis of figures.

Vnder[stand] also that in multiplicacio*u*n, divisio*u*n, and of rootis the extraccio*u*n, competently me may leve a mydel space betwix .2. ordres of figures, that me may write there what is come of addyng other with*e*-drawyng, lest any thynge sholde be ou*er*-hippe*d*e and sette out of mynde.

ter VII. Division.

Definition of dyvysion.

For to dyvyde oo nombre by a-nother, it is of .2. nombres p*ro*pose*d*e, It is forto depart the moder nombre into as many p*ar*tis as ben of vnytees in the lasse nombre. And note wele that in makynge of dyvysio*u*n ther ben .3. nombres necessary: Dividend, Divisor, Quocient, that is to seye, the nombre to be dyvydede; the nombre dyvydyng and the nombre exeant, other how oft, or quocient. Ay shalle the nombre that is to be dyvydede be more, other at the lest evene w*ith* the nombre the dyvysere, yf the nombre shalle be made by hole nombres. How to set downe yor sum. Therfor yf thow wolt any nombre dyvyde, write the nombre to be dyvydede in þe ou*er*er bordure by his differences, the dyviser*e* in the lower ordure by his differences, so that the last of the dyviser be vnder the last of the nombre to be dyvyde, the next last vnder the next last, and so of the others, yf it may competently be done; An example as here:

The residue		2	7
The quotient			5
To be dyvydede	3	4	2
The dyvyser		6	3

Examples.

Residuu*m*			8				2	7		2	6
Quociens		2	1	2	2			5			9
Diuidend*us*	6	8	0	6	6	3	4	2	3	3	2
Diuiser	3	2		3			6	3		3	4

When the last of the divisor must not be set below the last of the dividend. And ther ben .2. causes whan the last figure may not be sette vnder the last, other that the last of the lower nombre may not be w*ith*-draw of the last of the ou*er*er nombre for it is lasse than the lower, other how be it, that it myght be w*ith*-draw as for hym-self fro the ou*er*er the remenaunt may not so oft of them above,

other yf þe last of the lower be even to the figure above his hede, and þe next last oþ*er* the figure be-fore þ*a*t be more þan the figure above sette. Fol. 53[2]. *These so ordeynede*e*, me most wirch*e* from the last figure of þe nombre of the dyvyser, and se how oft it may be w*ith*-draw of and fro the figure aboue his hede, namly so that the remen*au*nt may be take of so oft, and to se the residue as here:— *[How to begin.]*

The residue		2	6
The quocient			9
To be dyvydede	3	3	2
The dyvyser		3	4

[An example.]

And note wele that me may not with*e*-draw more than .9. tymes nether lasse than ones. Therfor se how oft þe figures of the lower ordre may be w*ith*-draw fro the figures of the oue*r*er, and the nombre that shew*ith* þe q*u*ocient most be writ oue*r* the hede of þat figure, vnder the which*e* the first figure is, of the dyviser; *[Where to set the quocient.]* And by that figure me most with*e*-draw all*e* oþ*er* figures of the lower ordir and that of the figures aboue thair*e* hedis. This so don*e*, me most sette forward*e* þe figures of the diuiser by o difference toward*es* the right hond*e* and worch*e* as before; and thus:— *[Examples.]*

Residuu*m*											.	1	2
quo*ciens*				6	5	4				2	0	0	4
Diuidend*us*	3	5	5	1	2	2	8	8	6	3	7	0	4
Diuisor		5	4	3			4	4	2	3			

The quocient				6	5	4
To be dyvydede	3	5	5	1	2	2
The dyvyser		5	4	3		

[A special case.]

And yf it happ*e* after þe settyng forward*e* of the fig*ur*es þ*a*t þe last of the divisor may not so oft be w*ith*draw of the fig*ur*e above his hede, above þat fig*ur*e vnder the which*e* the first

of the diuiser is writ me most sette a cifre in ordre of the nombre quocient, and sette the fig*ur*es forward*e* as be-fore be o difference alone, and so me shall*e* do in all*e* nombres to be dyvided*e*, for where the dyviser may not be w*ith*-draw me most sette there a cifre, and sette forward*e* the figures; as here:—

The residue						1	2
The quocient				2	0	0	4
To be dyvyded*e*	8	8	6	3	7	0	4
The dyvyser	4	4	2	3			

Another example.

And me shall*e* not cesse fro such*e* settyng of fig*ur*es forward*e*, nether of settyng*e* of þe quocient into the dyviser, neþ*er* of subt*ra*ccio*u*n of the dyvyser, till*e* the first of the dyvyser be w*ith*-draw fro þe first to be divided*e*. The which*e* don*e*, or ought, [17] oþ*er* nought shall*e* remayne: and yf it be ought, [17] kepe it in the tables, And eu*er* vny it to þe diviser. And yf þ*o*u wilt wete how many vnytees of þe divisio*u*n Fol. 53[3]. *wol growe to the nombre of the divisere, the nombre quocient wol shewe it: and whan such*e* divisio*u*n is made, and þ*o*u lust p*ro*ve yf thow have wele done or no, Multiplie the quocient by the diviser, And the same fig*ur*es wolle come ayene that thow haddest bifore and none other. And yf ought be residue, than w*ith* addicio*u*n therof shall*e* come the same figures: And so multiplicacio*u*n p*ro*vith*e* divisio*u*n, and dyvisio*u*n multiplicacio*u*n: as thus, yf a multiplicacio*u*n be made, divide it by the multipliant, and the nombre quocient wol shewe the nombre that was to be multiplied*e*, *et*c.

What the quotient showeth.

How to prove your division, or multiplication.

ter VIII. Progression.

Definition of Progression.

Progressio*u*n is of nombre after egall*e* excesse fro oone or tweyn*e* take ag*r*egacio*u*n. of p*ro*gressio*u*n one is naturell*e* or co*n*tynuell*e*, þ*a*t oþ*er* broken and discontynuell*e*.

Naturell it is, whan me begynnethe with one, and kepethe ordure ouerlepyng one; as .1. 2. 3. 4. 5. 6., etc., so þat the nombre folowynge passithe the other be-fore in one.

Natural Progression.

Broken it is, whan me lepithe fro o nombre tille another, and kepithe not the contynuel ordire; as 1. 3. 5. 7. 9, etc. Ay me may begynne with .2., as þus; .2. 4. 6. 8., etc., and the nombre folowyng passethe the others by-fore by .2. And note wele, that naturelle progressioun ay begynnethe with one, and Intercise or broken progressioun, omwhile begynnythe with one, omwhile with twayne. Of progressioun naturell .2. rules ther be yove, of the whiche the first is this; whan the progressioun naturelle endithe in even nombre, by the half therof multiplie þe next totalle ouerere nombre; Example of grace: .1. 2. 3. 4. Multiplie .5. by .2. and so .10. comethe of, that is the totalle nombre þerof. The seconde rule is suche, whan the progressioun naturelle endithe in nombre ode. Take the more porcioun of the oddes, and multiplie therby the totalle nombre. Example of grace 1. 2. 3. 4. 5., multiplie .5. by .3, and thryes .5. shalle be resultant. so the nombre totalle is .15.

Broken Progression.

The 1st rule for Natural Progression.

The second rule.

The first rule of Broken Progression.

Of progressioun intercise, ther ben also .2. [18] rules; and þe first is þis: Whan the Intercise progression endithe in even nombre by half therof multiplie the next nombre to þat halfe as .2. [18] 4. 6. Multiplie .4. by .3. so þat is thryes .4., and .12. the nombre of alle the progressioun, wolle folow. The seconde rule is this: whan the progressioun interscise endithe in ode, take þe more porcioun of alle þe nombre, Fol. 53[4]. *and multiplie by hym-selfe; as .1. 3. 5. Multiplie .3. by hym-selfe, and þe some of alle wolle be .9., etc.

The second rule.

ter IX. Extraction of Roots.

The preamble of the Extraction of rotis.

Here folowithe the extraccioun of rotis, and first in nombre quadrates. Wherfor me shalle se what is

a nombre quadrat, and what is the rote of a nombre quadrat, and what it is to draw out the rote of a nombre. And before other note this divisio*u*n: Linear, superficial, and solid numbers. Of nombres one is lyneal, anoþ*er* sup*er*fic*ia*ll*e*, anoþ*er* quadrat, anoþ*er* cubik*e* or hoole. lyneal is that þat is considred*e* after the p*ro*cesse, havyng*e* no respect to the direccio*u*n of nombre in nombre, As a lyne hath*e* but one dymensio*u*n that is to sey after the lenght*e*. Superficial numbers. Nombre sup*er*ficial is þ*a*t cometh*e* of ledyng*e* of oo nombre into a-nother, wherfor it is called*e* sup*er*ficial, for it hath*e* .2. nombres notyng or mesuryng*e* hym, as a sup*er*ficiall*e* thyng*e* hath*e* .2. dimensions, þ*a*t is to sey lengt*he* and brede. Square numbers. And for-bycause a nombre may be had*e* in a-nother by .2. man*er*s, þ*a*t is to sey other in hym-self*e*, oþ*er* in anoþ*er*, Vnderstond*e* yf it be had in hym-self, It is a quadrat. ffor dyvisio*u*n write by vnytes, hath*e* .4. sides even as a quadrangill*e*. and yf the nombre be had*e* in a-noþ*er*, the nombre is sup*er*ficiel and not quadrat, as .2. had*e* in .3. maketh*e* .6. that is þe first nombre sup*er*ficiell*e*; wherfor it is open þat all*e* nombre quadrat is sup*er*ficiel, and not co*n*uertid*e*. The root of a square number. The rote of a nombre quadrat is þat nombre that is had of hym-self, as twies .2. makith*e* 4. and .4. is the first nombre quadrat, and 2. is his rote. 9. 8. 7. 6. 5. 4. 3. 2. 1. / The rote of the more quadrat .3. 1. 4. 2. 6. Notes of some examples of square roots here interpolated. The most nombre quadrat 9. 8. 7. 5. 9. 3. 4. 7. 6. / the remenent ou*er* the quadrat .6. 0. 8. 4. 5. / The first caas of nombre quadrat .5. 4. 7. 5. 6. The rote .2. 3. 4. The second*e* caas .3. 8. 4. 5. The rote .6. 2. The third*e* caas .2. 8. 1. 9. The rote .5. 3. The .4. caas .3. 2. 1. The rote .1. 7. / The 5. caas .9. 1. 2. 0. 4. / The rote 3. 0. 2. Solid numbers. The solide nombre or cubik*e* is þat þ*a*t comythe of double ledyng of nombre in nombre; Three dimensions of solids. And it is cleped*e* a solid*e* body that hath*e* þ*er*-in .3 [dimensions] þat is to sey, length*e*, brede, and thiknesse. so þ*a*t nombre hath*e* .3. nombres to be brought forth*e* in hym. But nombre may be had*e* twies in nombre, for other it is had*e* in hym-self*e*, oþ*er* in a-noþ*er*.

Cubic numbers. If a nombre be had*e* twies in hym-self, oþ*er* ones in his

quadrat, þat is the same, þat a cubike Fol. 54. *is, And is the same that is solide. And yf a nombre twies be hade in a-noþer, the nombre is clepede solide and not cubike, as twies .3. and þat .2. makithe .12. All cubics are solid numbers. Wherfor it is opyne that alle cubike nombre is solide, and not conuertide. Cubike is þat nombre þat comythe of ledynge of hym-selfe twyes, or ones in his quadrat. And here-by it is open that o nombre is the roote of a quadrat and of a cubike. Natheles the same nombre is not quadrat and cubike. No number may be both linear and solid. Opyne it is also that alle nombres may be a rote to a quadrat and cubike, but not alle nombre quadrat or cubike. Therfor sithen þe ledynge of vnyte in hym-self ones or twies nought comethe but vnytes, Seithe Boice in Arsemetrike, Unity is not a number. that vnyte potencially is al nombre, and none in act. And vndirstonde wele also that betwix euery .2. quadrates ther is a meene proporcionalle, Example of square roots. That is openede thus; lede the rote of o quadrat into the rote of the oþer quadrat, and þan wolle þe meene shew.

Residuum			0					4			0					0	
Quadrande	4	3	5	6	3	0	2	9	1	7	4	2	4	1	9	3	6
Duplum	1	2			1	0			2		6				[8]	19	
Subduplum		6		6		5		5	1		3		2		4		4

A note on medial proportionals. Also betwix the next .2. cubikis, me may fynde a double meene, that is to sey a more meene and a lesse. The more meene thus, as to bringe the rote of the lesse into a quadrat of the more. The lesse thus, If the rote of the more be brought Into the quadrat of the lesse.

ter X. Extraction of Square Root.

To[20] draw a rote of the nombre quadrat it is What-euer nombre be proposede to fynde his rote and to se yf it be quadrat. To find a square root. And yf it be not quadrat the rote of the most quadrat fynde out, vnder the nombre proposede. Therfor yf

thow wilt the rote of any quadrat nombre draw out, write the nombre by his differences, and compt the nombre of the figures, and wete yf it be od*e* or even. And yf it be even, than most thow begynne worche vnder the last save one. And yf it be od*e* w*ith* the last; and forto sey it shortly, al-weyes fro the last od*e* me shall*e* begynne. Therfor vnder the last in an od place sette, me most fynd*e* a digit, the which*e* lad*e* in hym-self*e* it puttith*e* away that, þat is ou*er* his hede, oþ*er* as neigh*e* as me may: suche a digit found*e* and w*ith*draw fro his ou*er*er, me most double that digit and sette the double vnder the next figure toward*e* the right hond*e*, and his vnder double vnder hym. That done, than me most fy*n*d*e* a-noþ*er* digit vnder the next figure bifore the doubled*e*, *brought in double settethe a-way all*e* that is ou*er* his hede as to reward*e* of the doubled*e*: Than brought into hym-self settith*e* all away in respect of hym-self, other do it as nye as it may be do: other me may w*ith*-draw the digit [21][last] found*e*, and lede hym in double or double hym, and after in hym-self*e*; Than Ioyne to-geder the p*ro*duccion*e* of them bothe, So that the first figure of the last p*ro*duct be added*e* before the first of the first p*ro*duct*es*, the second*e* of the first, *et*c. and so forth*e*, subtrahe fro the totall*e* nombre in respect of þe digit.

Begin with the last word, that

Find the nearest square root of that number, subtract

and set the double one to the right.

the which is

Find the second figure by division.

Multiply the double by the second figure, and add after it the square of the second figure, and subtract.

Examples.

The residue														5	4	3	2
To be quadred*e*	4	1	2	0	9	1	5	1	3	9	9	0	0	5	4	3	2
The double		4	0				2		4			6		0			0
The vnder double	2		0		3	1		2		3	[3]		[0]		[0]		0

And if it hap þ*a*t no digit may be found*e*, Than sette a cifre vndre a cifre, and cesse not till*e* thow fynde a digit; and whan thow hast founde it to double it, ne þ*er* to sette the double*d*e forward*e* nether the vnder doubled*e*, Till thow fynde vndre the first figure a digit, the which*e* lad*e* in all*e* double, settyng away all*e* that is ou*er* hym in respect of the double*d*e: Than lede hym into hym-self*e*, and put a-way all*e* in regard*e* of hym, other as nygh*e* as thow mai. That done, other ought or nought wolle be the residue. If nought, than it shewith*e* that a nombre componed*e* was the quadrat, and his rote a digit last found*e* w*ith* vnder*e*-double other vndirdoubles, so that it be sette be-fore: And yf ought[22] remayn*e*, that shew*ith* that the nombre p*ro*posed*e* was not quadrat,[23] but a digit [last found with the subduple or subduples is]

Special cases.

Th*e* double residue.

This table is constructed for use in cube root sums, giving the value of ab.^{2}

1	2	3	4	5	6	7	8	9
2	8	12	16	20	24	28	32	36
3	18	27	36	45	54	63	72	81
4	32	48	64	80	96	112[24]	128	144
5	50	75	100	125	150	175	200	225
6	72	108	144	180	216	252	288	324
7	98	147	196	245	294	343	393	441
8	128	192	256	320	384	448	512	576
9	168	243	324	405	486	567	648	729[25]

The rote of the most quadrat conteyned*e* vndre the nombre p*ro*posed*e*.

How to prove the square root without of [illegible] remainder.

Th*e*for yf thow wilt p*ro*ve yf thow have wele do or no, Multiplie the digit last found*e* w*ith* the vnder-double o þ*er* vnder-doublis, and thow shalt fynde the same figures that thow haddest before; And so that nought be the Fol. 55. *residue. And yf thow have

any residue, than w*ith* the addicio*u*n þ*er*of that is res*er*ued*e* w*ith*-out in thy table, thow shalt fynd*e* thi first figures as thow haddest them before, *et*c.

ter XI. Extraction of Cube Root.

Definition of a cubic number and a cube root.

Heere folowith*e* the extraccio*u*n of rotis in cubik*e* nombres; wher-for me most se what is a nombre cubik*e*, and what is his roote, And what is the extraccio*u*n of a rote. A nombre cubik*e* it is, as it is before declared*e*, that comeht*e* of ledyng of any nombre twies in hym-self*e*, other ones in his quadrat. The rote of a nombre cubik*e* is the nombre that is twies had*e* in hy*m*-self*e*, or ones in his quadrat. Wher-thurgh*e* it is open, that eu*er*y nombre quadrat or cubik*e* have the same rote, as it is seid*e* before. And forto draw out the rote of a cubik*e*, It is first to fynd*e* þe nombr*e* p*ro*posed*e* yf it be a cubik*e*; And yf it be not, than thow most make extraccio*u*n of his rote of the most cubik*e* vndre the nombre p*ro*posid*e* his rote found*e*. Therfor p*ro*posed*e* some nombre, whos cubical rote þ*o*u woldest draw out;

Mark off the places in threes.

First thow most compt the figures by fourthes, that is to sey in the place of thousand*es*;

Find the first digit;

And vnder the last thousand*e* place, thow most fynde a digit, the which*e* lad*e* in hym-self cubikly puttith*e* a-way that þat is ou*er* his hede as in respect of hym, other as nygh*e* as thow maist.

treble it and place it under the next but one, and multiply by the digit.

That done, thow most trebill*e* the digit, and that triplat is to be put vnder the .3. next figure toward*e* the right hond*e*, And the vnder-trebill*e* vnder the trebill*e*;

Then find the second digit.

Than me most fynd*e* a digit vndre the next figure bifore the triplat, the which*e* w*ith* his vnder-trebill*e* had into a trebill*e*, aft*er*warde other vnder[trebille] [26] had in his p*ro*duccio*u*n, puttethe a-way all*e* that is ou*er* it in regard*e* of [27] [the triplat. Then lade in hymself puttithe away that þat is over his hede as in respect of hym, other as nyghe as thou maist:]

Multiply the first triplate

and the second digit, That done, thow most trebill*e* the digit ayene, and the triplat is to be sette vnder the next .3. figure as before, And the vnder-trebill*e* vnder the trebill*e*: and than most thow sette forward*e* the first triplat w*ith* his vndre-trebill*e* by .2. differences. And than most thow fynde a digit vnder the next figure before the triplat, the which*e* with*e* his vnder-t*r*iplat had in his triplat afterwarde, vnder-treblis lad in p*ro*duct Fol. 55 *b*. *It sittethe a-way all that is ou*er* his hede in respect of the triplat than had in hym-self cubikly, [28] or as nygh*e* as ye may.

Examples.

Residuu*m*							5						4		1	0	1	9	
Cubicandu*s*	8	3	6	5	4	3	2	3	0	0	7	6	7	1	1	6	6	7	
Triplum			6	0							1	8					4		
Subt*r*iplu*m*	2			0			[3]			6			7		2			2	

Continue this process till the first figure is reached.

Nother me shall*e* not cesse of the fyndyng*e* of that digit, neither of his triplacio*u*n, neþ*er* of the triplat-is [29]anterioraci*ou*n, that is to sey, settyng forward*e* by .2. differences, Ne therof the vndre-triple to be put vndre the triple, Nether of the multiplicacio*u*n þ*er*of, Neither of the subtraccio*u*n, till*e* it come to the first figure, vnder the which*e* is a digitall*e* nombre to be found*e*, the which*e* with*e* his vndre-treblis most be had*e* in tribles, After-ward*e* w*ith*out vnder-treblis to be had*e* into produccio*u*n, settyng away all*e* that is ou*er* the hed*e* of the triplat nombre, After had into hymself*e* cubikly, sette alle-way that is ou*er* hym.

Examples.

To be cubiced*e*	1	7	2	8	3	2	7	6	8
The triple			3	2				9	
The vnder triple			1	2		[3]		3	3

Also note wele that the p*ro*ducc*i*on comyng*e* of the ledyng of a digite found*e* [30] me may adde to, and also w*ith*-draw fro of the totall*e* nombre sette above that digit so found*e*. [31] The residue

hat done, ought or nought most be the residue. If it be nought, It is

open that the nombre p*ro*pose*d*e was a cubik*e* nombre, And his rote a digit founde last w*ith* the vnder-triples: If the rote therof wex bad*e* in hym-self*e*, and afterward*e* p*ro*duct they shall*e* make the first fig*ur*es. And yf ought be in residue, kepe that w*ith*out in the table; and it is open*e* that the nombre was not a cubik*e*. but a digit last founde w*ith* the vndirtriplis is rote of the most cubik*e* vndre the nombre p*ro*pose*d*e conteyned*e*, the which*e* rote yf it be had*e* in hym-self. And afterward*e* in a p*ro*duct of that shall*e* growe the most cubik*e* vndre the nombre p*ro*pose*d*e conteyned*e*, And yf that be added*e* to a cubik*e* the residue res*er*ued*e* in the table, woll*e* make the same figures that ye had*e* first. Special

56. *And yf no digit after the anterioracio*u*n [32] may not be found*e*, than put there a cifre vndre a cifre vndir the third*e* figure, And put forward*e* þe fig*ur*es. Note also wele that yf in the nombre p*ro*pose*d*e ther ben no place of thowsand*es*, me most begynne vnder the first figure in the extraccio*u*n of the rote. some vsen forto distingue the nombre by threes, and ay begynne forto wirch*e* vndre the first of the last t*er*nary other unco*m*plete nombre, the which*e* maner of op*er*acio*u*n accordeth*e* w*ith* that before.

Examples.

The residue							0						1	1
The cubicand*us*	8	0	0	0	0	0	0	8	2	4	2	4	1	9
The triple			[33]	0	0					6				
The vndert*r*iple	[2]			0	0			2			6	2		

And this at this tyme suffiseth*e* in extraccio*u*n of nombres quadrat or cubik*es* et*c*.

of Numbers, &c.

A table of numbers; probably from the Abacus.

1 2 3 4 5

one. x. an. hundred*e* / a thowsand*e* / x. thowsand*e* /

6	7
An hundred*e* thowsand*e* /	A thowsand*e* tymes a thowsand*e* /

x. thousand*e* tymes a thousand*e* / An hundred*e* thousand*e* tymes a thousand*e* A thousand*e* thousand*e* tymes a thousand*e* / this is the x place *et*c.

[Ende.]

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1. MS. Materiall*e*.
2. MS. Formall*e*.
3. ‘the’ in MS.
4. ‘be’ in MS.
5. ‘and’ in MS.
6. ‘is’ in MS.
7. 6 in MS.
8. 0 in MS.
9. 2 in MS.
10. *sic*.
11. ‘And’ inserted in MS.
12. ‘4 the’ inserted in MS.
13. ‘to’ in MS.
14. ‘that’ repeated in MS.
15. ‘1’ in MS.
16. Blank in MS.
17. ‘nought’ in MS.
18. 3 written for 2 in MS.
19. 7 in MS.
20. runs on in MS.

21. ‘so’ in MS.

22. ‘nought’ in MS.

23. MS. adds here: ‘wher-vpon*e* se the table in the next side of the next leef*e*.’

24. 110 in MS.

25. 0 in MS.

26. double in MS.

27. ‘it hym-self*e*’ in MS.

28. MS. adds here: ‘it settethe a-way all*e* his respect.’

29. ‘aucterioracio*u*n’ in MS.

30. MS. adds here: ’w*ith* an vndre-triple / other of an vndre-triple in a triple or triplat is And after-ward*e* w*ith* out vndre-triple other vndre-triplis in the p*ro*duct and ayene that p*ro*duct that comeths of the ledynge of a digit found*e* in hym-self*e* cubicall*e*’ /

31. MS. adds here: ‘as ther had be a divisio*u*n made as it is openede before.’

32. MS. anteriocacio*u*n.

33. 4 in MS.

Accomptynge by counters.

The original text was printed as a single continuous paragraph, with no break between speakers; all examples were shown inline. It has been broken up for this e-text.

116 *b*.

* ¶ The seconde dialoge of accomptynge by counters.

Mayster.

Nowe that you haue learned the commen kyndes of Arithmetyke with the penne, you shall se the same art in cou*n*ters: whiche feate doth not only serue for them that can not write and rede, but also for them that can do bothe, but haue not at some tymes theyr penne or tables redye with them. This sorte is in two fourmes co*m*menly. The one by lynes, and the other without lynes: in that y^{t} hath lynes, the lynes do stande for the order of places: and in y^{t} that hath no lynes, there must be sette in theyr stede so many counters as shall nede, for eche lyne one, and they shall supplye the stede of the lynes.

S. By examples I shuld better p*er*ceaue your meanynge.

M. For example of the 117 *a* ly*nes:

```
      1   0  0  0  0  0
      1   0  0  0  0
X     1   0  0  0
      1   0  0
      1   0
      1
```

Lo here you se .vi. lynes whiche stande for syxe places so that the nethermost standeth for y^{e} fyrst place, and the next aboue it, for the second: and so vpward tyll you come to the hyghest, which is the syxte lyne, and standeth for the syxte place.

Now what is the valewe of euery place or lyne, you may perceaue by the figures whiche I haue set on them, which is accordynge as you learned before in the Numeration of figures by the penne: for the fyrste place is the place of vnities or ones, and euery counter set in that lyne betokeneth but one: *and* the seconde lyne is the place of 10, for euery counter there, standeth for 10. The thyrd lyne the place of hundredes: the fourth of thousandes: *and* so forth.

Numeration.

S. Syr I do perceaue that the same order is here of lynes, as was in the other figures 117 *b.* *by places, so that you shall not nede longer to stande about Numeration, excepte there be any other difference.

M. Yf you do vndersta*n*de it, then how wyll you set 1543?

X 1
5
4
3

S. Thus, as I suppose.

M. You haue set y^e^ places truely, but your figures be not mete for this vse: for the metest figure in this behalfe, is the figure of a cou*n*ter round, as you se here, where I haue expressed that same summe.

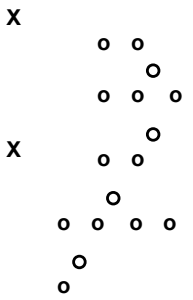
× o
o

o o o o
o o o

S. So that you haue not one figure for 2, nor 3, nor 4, and so forth, but as many digettes as you haue, you set in the lowest lyne: and for euery 10 you set one in the second line: and so of other. But I know not by what reason you set that one counter for 500 betwene two lynes.

M. you shall remember this, that when so euer you nede to set downe 5, 50, or 500, or 5000, or so forth any other nomber, whose numerator 118 *a.* *is 5, you shall set one counter for it, in the next space aboue the lyne that it hath his denomination of, as in this example of that 500, bycause the numerator is 5, it must

be set in a voyd space: and bycause the denominator is hundred, I knowe that his place is the voyde space next aboue hundredes, that is to say, aboue the thyrd lyne. And farther you shall marke, that in all workynge by this sorte, yf you shall sette downe any summe betwene 4 and 10, for the fyrste parte of that nomber you shall set downe 5, & then so many counters more, as there reste no*m*bers aboue 5. And this is true bothe of digettes and articles. And for example I wyll set downe this su*m*me 287965,

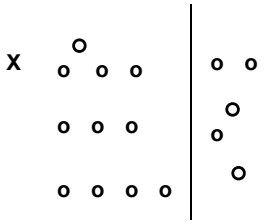


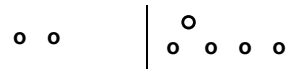
which su*m*me yf you marke well, you nede none other exa*m*ples for to lerne the numeration of 118 *b.* *this forme. But this shal you marke, that as you dyd in the other kynde of arithmetike, set a pricke in the places of thousa*n*des, in this worke you shall sette a starre, as you se here.

on on the Counting Board.

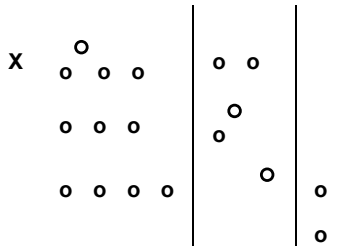
S. Then I perceave numeration, but I praye you, howe shall I do in this arte to adde two summes or more together?

M. The easyest way in this arte is, to adde but 2 su*m*mes at ones together: how be it you may adde more, as I wyll tell you anone. Therfore when you wyll adde two su*m*mes, you shall fyrst set downe one of them, it forseth not whiche, *and* then by it drawe a lyne crosse the other lynes. And afterward set downe the other su*m*me, so that that lyne may be betwene them, as yf you wolde adde 2659 to 8342, you must set your su*m*mes as you se here.

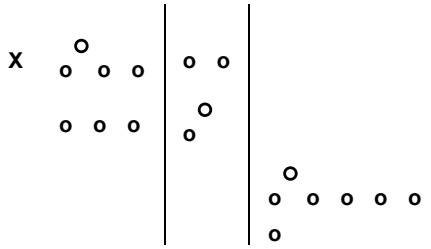




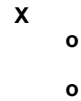
And then yf you lyst, you 119 *a.* *may adde the one to the other in the same place, or els you may adde them both together in a newe place: which waye, bycause it is moste playnest, I wyll showe you fyrst. Therfore wyl I begynne at the vnites, whiche in the fyrst su*m*me is but 2, *and* in y^e^ second su*m*me 9, that maketh 11, those do I take vp, and for them I set 11 in the new roume, thus,



Then do I take vp all y^e^ articles vnder a hundred, which in the fyrst su*m*me are 40, and in the second summe 50, that maketh 90: or you may saye better, that in the fyrste summe there are 4 articles of 10, and in the seconde summe 5, which make 9, but then take hede that you sette them in theyr 119 *b.* *ryght lynes as you se here.



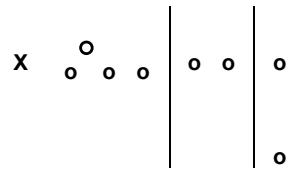
Where I haue taken awaye 40 fro*m* the fyrste su*m*me, and 50 from y^e^ second, and in theyr stede I haue set 90 in the thyrde, whiche I haue set playnely y^t^ you myght well perceaue it: how be it seynge that 90 with the 10 that was in y^e^ thyrd roume all redy, doth make 100, I myghte better for those 6 cou*n*ters set 1 in the thyrde lyne, thus:



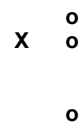
For it is all one summe as you may se, but it is beste, neuer to set 5 cou*n*ters in any line, for that may be done with 1 cou*n*ter in a hygher place.

S. I iudge that good reaso*n*, for many are vnnedefull, where one wyll serue.

M. Well, then 120 *a.* *wyll I adde forth of hundredes: I fynde 3 in the fyrste summe, and 6 in the seconde, whiche make 900, them do I take vp *and* set in the thyrd roume where is one hundred all redy, to whiche I put 900, and it wyll be 1000, therfore I set one cou*n*ter in the fourth lyne for them all, as you se here.



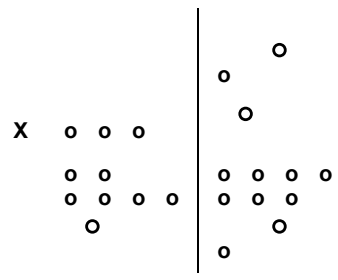
Then adde I y^e^ thousandes together, whiche in the fyrst su*m*me are 8000, *and* in y^e^ second 2000, that maketh 10000: them do I take vp fro*m* those two places, and for them I set one counter in the fyfte lyne, and then appereth as you se,



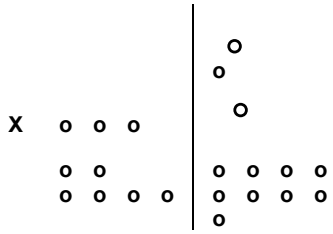
to be 11001, for so many doth amount of the addition of 8342 to 2659.

120 *b.* **S.* Syr, this I do perceave: but how shall I set one su*m*me to an other, not chaungynge them to a thyrde place?

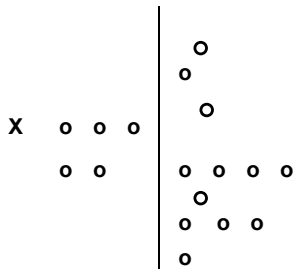
M. Marke well how I do it: I wyll adde together 65436, and 3245, whiche fyrste I set downe thus.



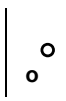
Then do I begynne with the smalest, which in the fyrst summe is 5, that do I take vp, and wold put to the other 5 in the seconde summe, sauynge that two counters can not be set in a voyd place of 5, but for them bothe I must set 1 in the seconde lyne, which is the place of 10, therfore I take vp the 5 of the fyrst su*m*me, *and* the 5 of the seco*n*de, and for them I set 1 in the seco*n*d lyne, 121 *a*. *as you se here.

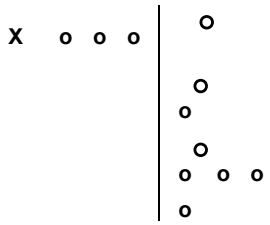


Then do I lyke wayes take vp the 4 counters of the fyrste su*m*me *and* seconde lyne (which make 40) and adde them to the 4 counters of the same lyne, in the second su*m*me, and it maketh 80, But as I sayde I maye not conueniently set aboue 4 cou*n*ters in one lyne, therfore to those 4 that I toke vp in the fyrst su*m*me, I take one also of the seconde su*m*me, and then haue I taken vp 50, for whiche 5 counters I sette downe one in the space ouer y^e^ second lyne, as here doth appere.

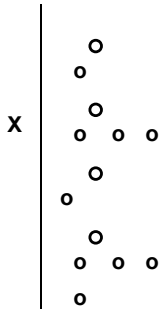


121 *b* *and then is there 80, as well w^t^ those 4 counters, as yf I had set downe y^e^ other 4 also. Now do I take the 200 in the fyrste su*m*me, and adde them to the 400 in the seconde summe, and it maketh 600, therfore I take vp the 2 counters in the fyrste summe, and 3 of them in the seconde summe, and for them 5 I set 1 in y^e^ space aboue, thus.

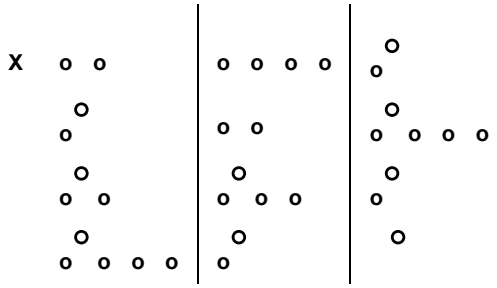




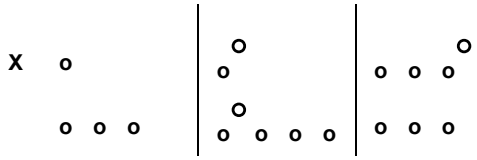
Then I take y^e 3000 in y^e fyrste su*m*me, vnto whiche there are none in the second summe agreynge, therfore I do onely remoue those 3 counters from the fyrste summe into the seconde, as here doth appere.

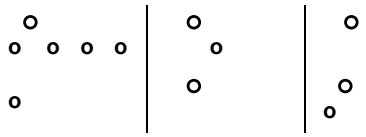


122 *a* *And so you see the hole su*m*me, that amou*n*teth of the addytio*n* of 65436 with 3245 to be 6868[1]. And yf you haue marked these two exa*m*ples well, you nede no farther enstructio*n* in Addition of 2 only summes: but yf you haue more then two summes to adde, you may adde them thus.



Fyrst adde two of them, and then adde the thyrde, and y^e fourth, or more yf there be so many: as yf I wolde adde 2679 with 4286 and 1391. Fyrste I adde the two fyrste summes thus. 122 *b.* *And then I adde the thyrde thereto thus.



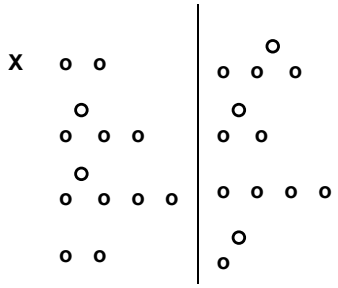


And so of more yf you haue them.

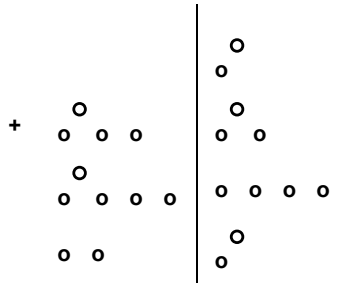
ction on the Counting Board.

S. Nowe I thynke beste that you passe forth to Subtraction, except there be any wayes to examyn this maner of Addition, then I thynke that were good to be knowen nexte.

M. There is the same profe here that is in the other Addition by the penne, I meane Subtraction, for that onely is a sure waye: but consyderynge that Subtraction must be fyrste knowen, I wyl fyrste teache you the arte of Subtraction, and that by this example: I wolde subtracte 2892 out of 8746. These summes must I set downe as I dyd in Addition: but here it is best 116 *a* (*sic*). *to set the lesser no*m*ber fyrste, thus.

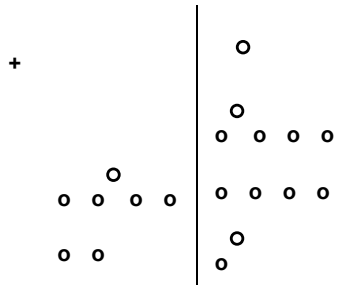


Then shall I begynne to subtracte the greatest nombres fyrste (contrary to the vse of the penne) y^{t} is the thousandes in this exa*m*ple: therfore I fynd amongest the thousandes 2, for which I withdrawe so many fro*m* the seconde summe (where are 8) and so remayneth there 6, as this exa*m*ple showeth.

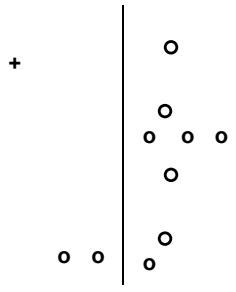


Then do I lyke wayes with the hundredes, of whiche in the fyrste summe 116 *b.* *I fynde 8, and is the seconde summe but 7, out of

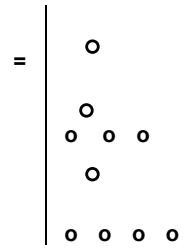
whiche I can not take 8, therfore thus muste I do: I muste loke how moche my summe dyffereth from 10, whiche I fynde here to be 2, then must I bate for my su*m*me of 800, one thousande, and set downe the excesse of hundredes, that is to saye 2, for so moche 100[0] is more then I shuld take vp. Therfore fro*m* the fyrste su*m*me I take that 800, and from the second su*m*me where are 6000, I take vp one thousande, and leue 5000; but then set I downe the 200 unto the 700 y^{t} are there all redye, and make them 900 thus.



Then come I to the articles of te*n*nes where in the fyrste su*m*me I fynde 90, 117 *a.* *and in the seconde su*m*me but only 40: Now consyderyng that 90 can not be bated from 40, I loke how moche y^{t} 90 doth dyffer from the next summe aboue it, that is 100 (or elles whiche is all to one effecte, I loke how moch 9 doth dyffer fro*m* 10) *and* I fynd it to be 1, then in the stede of that 90, I do take from the second summe 100: but consyderynge that it is 10 to moche, I set downe 1 in y^{e} nexte lyne beneth for it, as you se here.

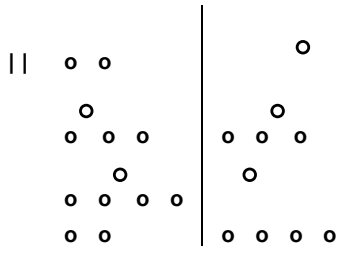


Sauynge that here I haue set one counter in y^{e} space in stede of 5 in y^{e} nexte lyne. And thus haue I subtracted all saue two, which I must bate from the 6 in the second summe, and there wyll remayne 4, thus.

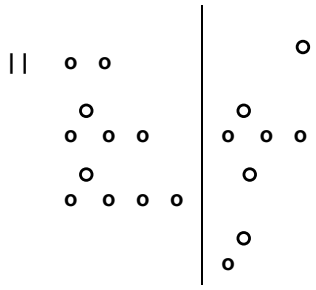


So y^t^ yf I subtracte 2892 fro*m* 8746, the remayner wyll be 5854,
117 *b.* *And that this is truely wrought, you maye proue by Addition: for yf you adde to this remayner the same su*m*me that you dyd subtracte, then wyll the formar su*m*me 8746 amount agayne.

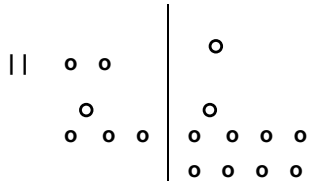
S. That wyll I proue: and fyrst I set the su*m*me that was subtracted, which was 2892, *and* the*n* the remayner 5854, thus.



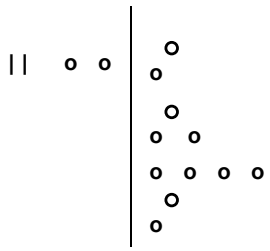
Then do I adde fyrst y^e^ 2 to 4, whiche maketh 6, so take I vp 5 of those counters, and in theyr stede I sette 1 in the space, as here appereth.



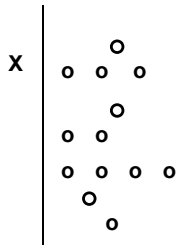
118 *a.* *Then do I adde the 90 nexte aboue to the 50, and it maketh 140, therfore I take vp those 6 counters, and for them I sette 1 to the hundredes in y^e^ thyrde lyne, *and* 4 in y^e^ second lyne, thus.



Then do I come to the hundredes, of whiche I fynde 8 in the fyrst summe, and 9 in y^e^ second, that maketh 1700, therfore I take vp those 9 counters, and in theyr stede I sette 1 in the .iiii. lyne, and 1 in the space nexte beneth, and 2 in the thyrde lyne, as you se here.



Then is there lefte in the fyrste summe but only 2000, whiche I shall take vp from thence, and set 118 *b.* *in the same lyne in y^e^ second su*m*me, to y^e^ one y^t^ is there all redy: *and* then wyll the hole su*m*me appere (as you may wel se) to be 8746, which was y^e^ fyrst grosse summe, *and* therfore I do perceaue, that I hadde well subtracted before.

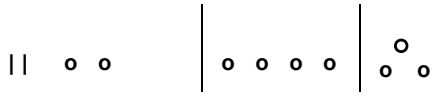


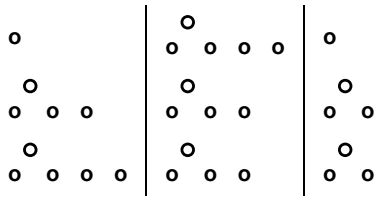
And thus you may se how Subtraction maye be tryed by Addition.

S. I perceaue the same order here w^t^ cou*n*ters, y^t^ I lerned before in figures.

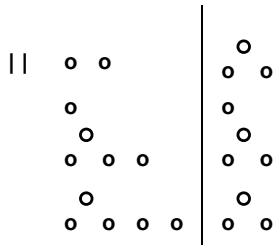
M. Then let me se howe can you trye Addition by Subtraction.

S. Fyrste I wyl set forth this exa*m*ple of Additio*n* where I haue added 2189 to 4988, and the hole su*m*me appereth to be 7177,

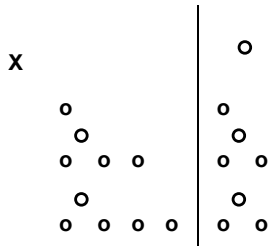




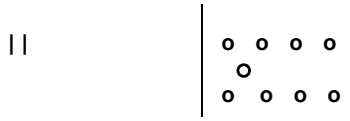
119 *a.* *Nowe to trye whether that su*m*me be well added or no, I wyll subtract one of the fyrst two su*m*mes from the thyrd, and yf I haue well done y^e^ remayner wyll be lyke that other su*m*me. As for example: I wyll subtracte the fyrste summe from the thyrde, whiche I set thus in theyr order.

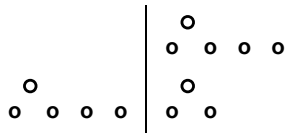


Then do I subtract 2000 of the fyrste summe fro*m* y^e^ second su*m*me, and then remayneth there 5000 thus.

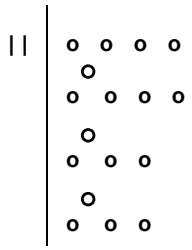


Then in the thyrd lyne, I subtract y^e^ 100 of the fyrste summe, fro*m* the second su*m*me, where is onely 100 also, and then in y^e^ thyrde lyne resteth nothyng. Then in the second lyne with his space ouer hym, I fynde 80, which I shuld subtract 119 *b.* *from the other su*m*me, then seyng there are but only 70 I must take it out of some hygher summe, which is here only 5000, therfore I take vp 5000, and seyng that it is to moch by 4920, I sette downe so many in the seconde roume, whiche with the 70 beynge there all redy do make 4990, & then the summes doth stande thus.



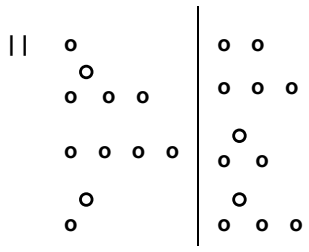


Yet remayneth there in the fyrst su*m*me 9, to be bated from the second summe, where in that place of vnities dothe appere only 7, then I muste bate a hygher su*m*me, that is to saye 10, but seynge that 10 is more then 9 (which I shulde abate) by 1, therfore shall I take vp one counter from the seconde lyne, *and* set downe the same in the fyrst 120 *a.* *or lowest lyne, as you se here.



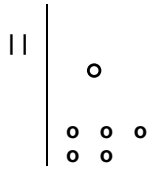
And so haue I ended this worke, *and* the su*m*me appereth to be y^e^ same, whiche was y^e^ seconde summe of my addition, and therfore I perceaue, I haue wel done.

M. To stande longer about this, it is but folye: excepte that this you maye also vnderstande, that many do begynne to subtracte with counters, not at the hyghest su*m*me, as I haue taught you, but at the nethermoste, as they do vse to adde: and when the summe to be abatyd, in any lyne appeareth greater then the other, then do they borowe one of the next hygher roume, as for example: yf they shuld abate 1846 from 2378, they set y^e^ summes thus.



120 *b.* *And fyrste they take 6 whiche is in the lower lyne, and his space from 8 in the same roumes, in y^e^ second su*m*me, and yet there remayneth 2 counters in the lowest lyne. Then in the

second lyne must 4 be subtracte from 7, and so remayneth there 3. Then 8 in the thyrde lyne and his space, from 3 of the second summe can not be, therfore do they bate it from a hygher roume, that is, from 1000, and bycause that 1000 is to moch by 200, therfore must I sette downe 200 in the thyrde lyne, after I haue taken vp 1000 from the fourth lyne: then is there yet 1000 in the fourth lyne of the fyrst summe, whiche yf I withdrawe from the seconde summe, then doth all y^e^ figures stande in this order.



So that (as you se) it differeth not greatly whether you begynne subtractio*n* at the hygher lynes, or at 121 *a.* *the lower. How be it, as some menne lyke the one waye beste, so some lyke the other: therfore you now knowyng bothe, may vse whiche you lyst.

lication by Counters.

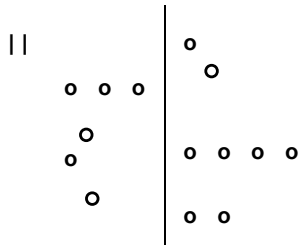
Multiplication

But now to ichynge Multiplicatio*n*: you shall set your no*m*bers in two roumes, as you dyd in those two other kyndes, but so that the multiplier be set in the fyrste roume. Then shall you begyn with the hyghest no*m*bers of y^e^ seconde roume, and multiply them fyrst after this sort. Take that ouermost lyne in your fyrst workynge, as yf it were the lowest lyne, setting on it some mouable marke, as you lyste, and loke how many counters be in hym, take them vp, and for them set downe the hole multyplyer, so many tymes as you toke vp counters, reckenyng, I saye that lyne for the vnites: *and* when you haue so done with the hygheest no*m*ber then come to the nexte lyne beneth, *and* do euen so with it, and so with y^e^ next, tyll you haue done all. And yf there be any nomber in a space, then for it 121 *b.* *shall you take y^e^ multiplyer 5 tymes, and then must you recken that lyne for the vnites whiche is nexte beneth that space: or els after a shorter way, you shall take only halfe the multyplyer, but then shall you take the lyne nexte aboue that space, for the lyne of vnites: but in suche workynge, yf chau*n*ce your multyplyer be an

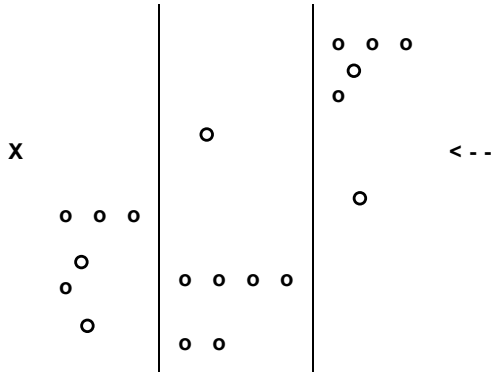
odde nomber, so that you can not take the halfe of it iustly, then muste you take the greater halfe, and set downe that, as if that it were the iuste halfe, and farther you shall set one cou*n*ter in the space beneth that line, which you recken for the lyne of vnities, or els only remoue forward the same that is to be multyplyed.

S. Yf you set forth an example hereto I thynke I shal perceaue you.

M. Take this exa*m*ple: I wold multiply 1542 by 365, therfore I set y^{e} nombers thus.



122 *a.* *Then fyrste I begynne at the 1000 in y^{e} hyghest roume, as yf it were y^{e} fyrst place, & I take it vp, settynge downe for it so often (that is ones) the multyplyer, which is 365, thus, as you se here:

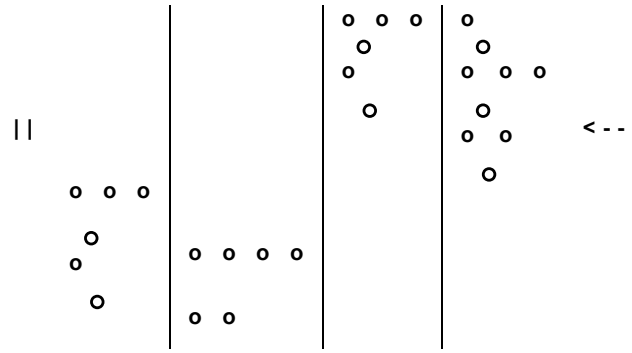


where for the one counter taken vp from the fourth lyne, I haue sette downe other 6, whiche make y^{e} su*m*me of the multyplyer, reckenynge that fourth lyne, as yf it were the fyrste: whiche thyng I haue marked by the hand set at the begynnyng of y^{e} same,

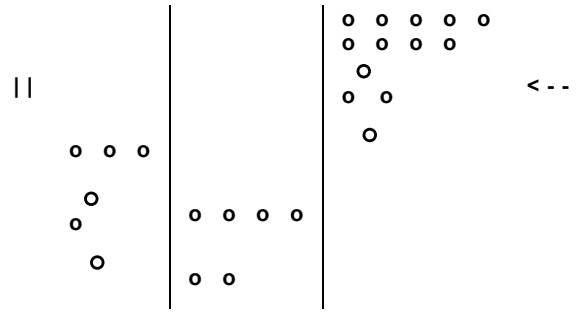
S. I perceaue this well: for in dede, this summe that you haue set downe is 365000, for so moche doth amount 122 *b.* *of 1000,

multiplyed by 365.

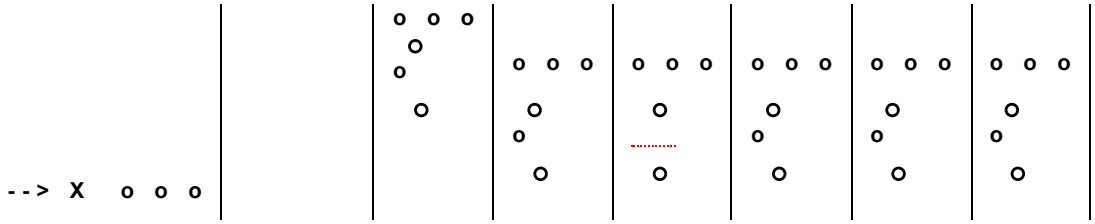
M. Well then to go forth, in the nexte space I fynde one counter which I remoue forward but take not vp, but do (as in such case I must) set downe the greater halfe of my multiplier (seyng it is an odde nomber) which is 182, *and* here I do styll let that fourth place stand, as yf it were y^e^ fyrst:

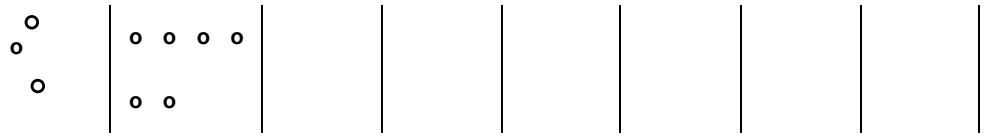


as in this fourme you se, where I haue set this multiplycation with y^e^ other: but for the ease of your vnderstandynge, I haue set a lytell lyne betwene them: now shulde they both in one summe stand thus.

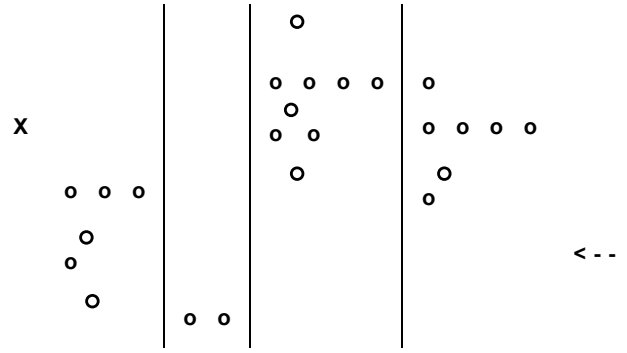


123 *a.* *Howe be it an other fourme to multyplye suche counters in space is this: Fyrst to remoue the fynger to the lyne nexte benethe y^e^ space, *and* then to take vp y^e^ counter, *and* to set downe y^e^ multiplyer .v. tymes, as here you se.

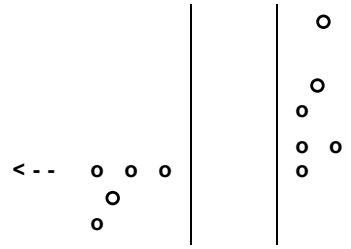


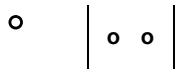


Which su*m*mes yf you do adde together into one su*m*me, you shal p*er*ceaue that it wyll be y^e^ same y^t^ appeareth of y^e^ other worki*n*g before, so that 123 *b.* *bothe sortes are to one entent, but as the other is much shorter, so this is playner to reason, for suche as haue had small exercyse in this arte. Not withstandynge you maye adde them in your mynde before you sette them downe, as in this exa*m*ple, you myghte haue sayde 5 tymes 300 is 1500, *and* 5 tymes 60 is 300, also 5 tymes 5 is 25, whiche all put together do make 1825, which you maye at one tyme set downe yf you lyste. But nowe to go forth, I must remoue the hand to the nexte counters, whiche are in the second lyne, and there must I take vp those 4 counters, settynge downe for them my multiplyer 4 tymes, whiche thynge other I maye do at 4 tymes seuerally, or elles I may gather that hole summe in my mynde fyrste, and then set it downe: as to saye 4 tymes 300 is 1200: 4 tymes 60 are 240: and 4 tymes 5 make 20: y^t^ is in all 1460, y^t^ shall I set downe also: as here you se.

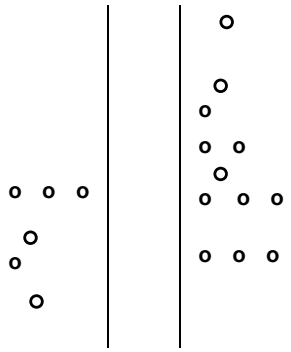


124 *a.* *whiche yf I ioyne in one summe with the formar nombers, it wyll appeare thus.



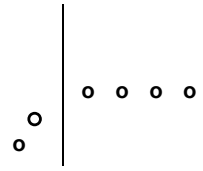


Then to ende this multiplycation, I remoue the fynger to the lowest lyne, where are onely 2, them do I take vp, and in theyr stede do I set downe twyse 365, that is 730, for which I set 124 *b.* *one in the space aboue the thyrd lyne for 500, and 2 more in the thyrd lyne with that one that is there all redye, and the reste in theyr order, *and* so haue I ended the hole summe thus.

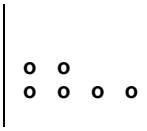


Wherby you se, that 1542 (which is the nomber of yeares syth Ch[r]ystes incarnation) beyng multyplyed by 365 (which is the nomber of dayes in one yeare) dothe amounte vnto 562830, which declareth y^e^ no*m*ber of daies sith Chrystes incarnatio*n* vnto the ende of 1542 [1] yeares. (besyde 385 dayes and 12 houres for lepe yeares).

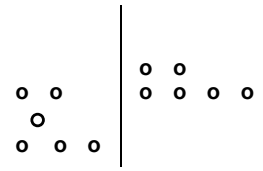
S. Now wyll I proue by an other exa*m*ple, as this: 40 labourers (after 6 d. y^e^ day for eche man) haue wrought 28 dayes, I wold 125 *a.* *know what theyr wages doth amou*n*t vnto: In this case muste I worke doublely: fyrst I must multyplye the nomber of the labourers by y^e^ wages of a man for one day, so wyll y^e^ charge of one daye amount: then secondarely shall I multyply that charge of one daye, by the hole nomber of dayes, *and* so wyll the hole summe appeare: fyrst therefore I shall set the su*m*mes thus.



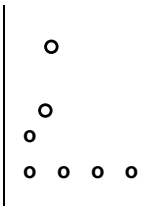
Where in the fyrste space is the multyplyer (y^{t} is one dayes wages for one man) *and* in the second space is set the nomber of the worke men to be multyplyed: the*n* saye I, 6 tymes 4 (reckenynge that second lyne as the lyne of vnites) maketh 24, for whiche summe I shulde set 2 counters in the thyrde lyne, and 4 in the seconde, therfore do I set 2 in the thyrde lyne, and let the 4 stand styll in the seconde lyne, thus.* 125 *b.*



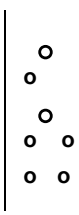
So apwereth the hole dayes wages to be 240d'. that is 20 s. Then do I multiply agayn the same summe by the no*m*ber of dayes and fyrste I sette the nombers, thus.



The*n* bycause there are counters in dyuers lynes, I shall begynne with the hyghest, and take them vp, settynge for them the multyplyer so many tymes, as I toke vp counters, y^{t} is twyse, then wyll y^{e} su*m*me stande thus.



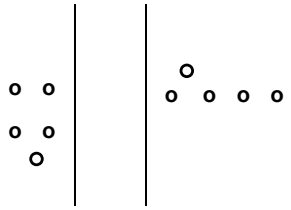
Then come I to y^{e} seconde lyne, and take vp those 4 cou*n*ters, settynge for them the multiplyer foure tymes, so wyll the hole summe appeare thus.* 126 *a.*



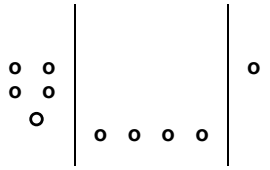
So is the hole wages of 40 workemen, for 28 dayes (after 6d'. eche daye for a man) 6720d'. that is 560 s. or 28 l'i.

ɔn on the Counting Board.

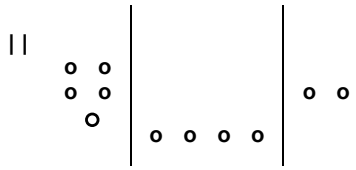
M. Now if you wold proue Multiplycation, the surest way is by Diuision. Dyuision: therfore wyll I ouer passe it tyll I haue taught you y^e^ arte of Diuision, whiche you shall worke thus. Fyrste sette downe the Diuisor for feare of forgettynge, and then set the nomber that shalbe deuided, at y^e^ ryghte syde, so farre from the diuisor, that the quotient may be set betwene them: as for example: Yf 225 shepe cost 45 l'i. what dyd euery shepe cost? To knowe this, I shulde diuide the hole summe, that is 45 l'i. by 225, but that can not be, therfore must I fyrste reduce that 45 l'i. into a lesser denomination, as into shyllynges: then I multiply 45 by 20, and it is 900, that summe shall I diuide by the nomber of 126 *b.* *shepe, whiche is 225, these two nombers therfore I sette thus.



Then begynne I at the hyghest lyne of the diuident, and seke how often I may haue the diuisor therin, and that maye I do 4 tymes, then say I, 4 tymes 2 are 8, whyche yf I take from 9, there resteth but 1, thus

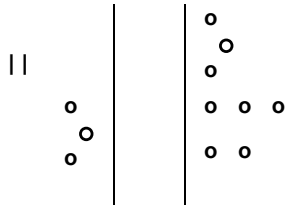


And bycause I founde the diuisor 4 tymes in the diuidente, I haue set (as you se) 4 in the myddle roume, which 127 *a.* *is the place of the quotient: but now must I take the reste of the diuisor as often out of the remayner: therfore come I to the seconde lyne of the diuisor, sayeng 2 foure tymes make 8, take 8 from 10, *and* there resteth 2, thus.



Then come I to the lowest nomber, which is 5, and multyply it 4 tymes, so is it 20, that take I from 20, and there remayneth nothynge, so that I se my quotient to be 4, whiche are in valewe shyllynges, for so was the diuident: and therby I knowe, that yf 225 shepe dyd coste 45 l'i. euery shepe coste 4 s.

S. This can I do, as you shall perceaue by this exa*m*ple: Yf 160 sowldyars do spende euery moneth 68 l'i. what spendeth eche man? Fyrst 127 *b.* *bycause I can not diuide the 68 by 160, therfore I wyll turne the pou*n*des into pennes by multiplicacio*n*, so shall there be 16320 d'. Nowe muste I diuide this su*m*me by the nomber of sowldyars, therfore I set the*m* i*n* order, thus.

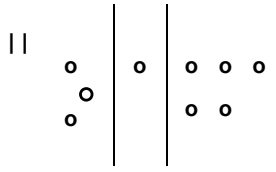


Then begyn I at the hyghest place of the diuidente, sekynge my diuisor there, whiche I fynde ones, Therfore set I 1 in the nether lyne.

M. Not in the nether line of the hole summe, but in the nether lyne of that worke, whiche is the thyrde lyne.

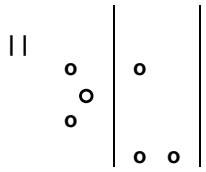
S. So standeth it with reason.

M. Then thus do they stande.* 128 *a.*



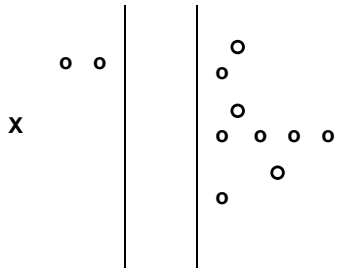
Then seke I agayne in the reste, how often I may fynde my diuisor, and I se that in the 300 I myghte fynde 100 thre tymes, but then the 60 wyll not be so often founde in 20, therfore I take 2 for my quotient: then take I 100 twyse from 300, and there

resteth 100, out of whiche with the 20 (that maketh 120) I may take 60 also twyse, and then standeth the nombers thus,



128 *b.* *where I haue sette the quotient 2 in the lowest lyne: So is euery sowldyars portion 102 d'. that is 8 s. 6 d'.

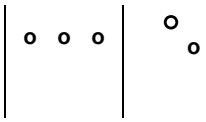
M. But yet bycause you shall perceaue iustly the reason of Diuision, it shall be good that you do set your diuisor styll agaynst those nombres fro*m* whiche you do take it: as by this example I wyll declare. Yf y^{e} purchace of 200 acres of ground dyd coste 290 l'i. what dyd one acre coste? Fyrst wyl I turne the poundes into pennes, so wyll there be 69600 d'· Then in settynge downe these nombers I shall do thus.



Fyrst set the diuident on the ryghte hande as it oughte, and then 129 *a.* *the diuisor on the lefte hande agaynst those nombers, fro*m* which I entende to take hym fyrst as here you se, wher I haue set the diuisor two lynes hygher the*n* is theyr owne place.

S. This is lyke the order of diuision by the penne.

M. Truth you say, and nowe must I set y^{e} quotient of this worke in the thyrde lyne, for that is the lyne of vnities in respecte to the diuisor in this worke. Then I seke howe often the diuisor maye be founde in the diuident, *and* that I fynde 3 tymes, then set I 3 in the thyrde lyne for the quotient, and take awaye that 60000 fro*m* the diuident, and farther I do set the diuisor one line lower, as yow se here.



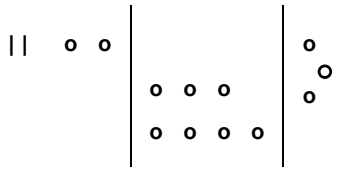
129 *b.* *And then seke I how often the diuisor wyll be taken from the nomber agaynste it, whiche wyll be 4 tymes and 1 remaynynge.

S. But what yf it chaunce that when the diuisor is so remoued, it can not be ones taken out of the diuident agaynste it?

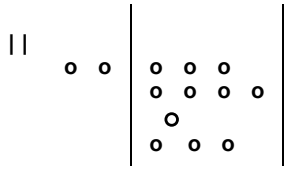
M. Then must the diuisor be set in an other line lower.

S. So was it in diuision by the penne, and therfore was there a cypher set in the quotient: but howe shall that be noted here?

M. Here nedeth no token, for the lynes do represente the places: onely loke that you set your quotient in that place which standeth for vnities in respecte of the diuisor: but now to returne to the example, I fynde the diuisor 4 tymes in the diuidente, and 1 remaynynge, for 4 tymes 2 make 8, which I take from 9, and there resteth 1, as this figure sheweth:



and in the myddle space for the quotient I set 4 in the seconde lyne, whiche is in this worke the place of vnities.* 130 *a.* Then remoue I y^e^ diuisor to the next lower line, and seke how often I may haue it in the dyuident, which I may do here 8 tymes iust, and nothynge remayne, as in this fourme,



where you may se that the hole quotient is 348 d', that is 29 s. wherby I knowe that so moche coste the purchace of one aker.

S. Now resteth the profes of Multiplycatio*n*, and also of Diuisio*n*.

M. Ther best profes are eche 130 *b.* *one by the other, for Multyplication is proued by Diuision, and Diuision by Multiplycation, as in the worke by the penne you learned.

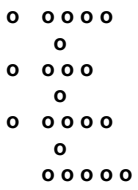
S. Yf that be all, you shall not nede to repete agayne that, y^{t} was sufficye*n*tly taughte all redye: and excepte you wyll teache me any other feate, here maye you make an ende of this arte I suppose.

M. So wyll I do as touchynge hole nomber, and as for broken nomber, I wyll not trouble your wytte with it, tyll you haue practised this so well, y^{t} you be full perfecte, so that you nede not to doubte in any poynte that I haue taught you, and thenne maye I boldly enstructe you in y^{e} arte of fractions or broken no*m*ber, wherin I wyll also showe you the reasons of all that you haue nowe learned. But yet before I make an ende, I wyll showe you the order of co*m*men castyng, wher in are bothe pennes, shyllynges, and poundes, procedynge by no grounded reason, but onely by a receaued 131 *a.* *fourme, and that dyuersly of dyuers men: for marchau*n*tes vse one fourme, and auditors an other:

ants' Casting Counters.

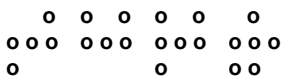
Auditours casting.

But fyrste for marchauntes fourme marke this example here,



in which I haue expressed this summe 198 l'i.[2] 19 s. 11 d'. So that you maye se that the lowest lyne serueth for pe*n*nes, the next aboue for shyllynges, the thyrde for poundes, and the fourth for scores of pou*n*des. And farther you maye se, that the space betwene pennes and shyllynges may receaue but one counter (as all other spaces lyke wayes do) and that one standeth in that place for 6 d'. Lyke wayes betwene the shyllynges *and* the pou*n*des, one cou*n*ter standeth for 10 s. And betwene the

poundes and 20 l'i. one counter standeth for 10 pou*n*des. But besyde those you maye see at the left syde of shyllynges, that one counter standeth alone, *and* betokeneth 5 s. 131 *b.* *So agaynste the poundes, that one cou*n*ter standeth for 5 l'i. And agaynst the 20 poundes, the one counter standeth for 5 score pou*n*des, that is 100 l'i. so that euery syde counter is 5 tymes so moch as one of them agaynst whiche he standeth. Auditors' ow for the accompt of auditors take this example.



where I haue expressed y^e^ same su*m*me 198 l'i. 19 s. 11 d'. But here you se the pe*n*nes stande toward y^e^ ryght hande, and the other encreasynge orderly towarde the lefte hande. Agayne you maye se, that auditours wyll make 2 lynes (yea and more) for pennes, shyllynges, *and* all other valewes, yf theyr summes extende therto. Also you se, that they set one counter at the ryght ende of eche rowe, whiche so set there standeth for 5 of that roume: and on 132 *a.* *the lefte corner of the rowe it sta*n*deth for 10, of y^e^ same row. But now yf you wold adde other subtracte after any of both those sortes, yf you marke y^e^ order of y^t^ other feate which I taught you, you may easely do the same here without moch teachynge: for in Additio*n* you must fyrst set downe one su*m*me and to the same set the other orderly, and lyke maner yf you haue many: but in Subtraction you must sette downe fyrst the greatest summe, and from it must you abate that other euery denominatio*n* from his dewe place.

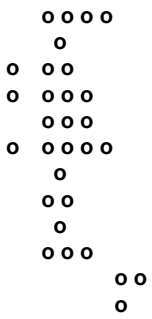
S. I do not doubte but with a lytell practise I shall attayne these bothe: but how shall I multiply and diuide after these fourmes?

M. You can not duely do none of both by these sortes, therfore in suche case, you must resort to your other artes.

S. Syr, yet I se not by these sortes how to expresse hu*n*dreddes, yf they excede one hundred, nother yet thousandes.

M. They that vse such accomptes that it excede 200 132 *b.* *in one summe, they sette no 5 at the lefte hande of the scores of

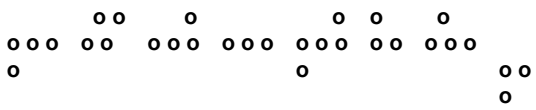
poundes, but they set all the hundredes in an other farther rowe *and* 500 at the lefte hand therof, and the thousandes they set in a farther rowe yet, *and* at the lefte syde therof they sette the 5000, and in the space ouer they sette the 10000, and in a hygher rowe 20000, whiche all I haue expressed in this exa*m*ple,



which is 97869 l'i. 12 s. 9 d' ob. q. for I had not told you before where, nother how you shuld set downe farthynges, which (as you se here) must be set in a voyde space sydelynge beneth the pennes: for q one counter: for ob. 2 counters: for ob. q. 3 counters: *and* more there can not be, for 4 farthynges 133 *a.* *do make 1 d'. which must be set in his dewe place.

ɔrs' Casting Counters.

And yf you desyre y^e^ same summe after audytors maner, lo here it is.



But in this thyng, you shall take this for suffycyent, and the reste you shall obserue as you maye se by the working of eche sorte: for the dyuers wittes of men haue inuented dyuers and sundry wayes almost vnnumerable. But one feate I shall teache you, whiche not only for the straungenes and secretnes is moche pleasaunt, but also for the good co*m*moditie of it ryghte worthy to be well marked. This feate hath ben vsed aboue 2000 yeares at the leaste, and yet was it neuer come*n*ly knowen, especyally in Englysshe it was neuer taughte yet. This is the arte of nombrynge on the hand, with diuers gestures of the fyngers, expressynge any summe conceaued in the 133 *b.* *mynde. And

fyrst to begynne, yf you wyll expresse any summe vnder 100, you shall expresse it with your lefte hande: and from 100 vnto 10000, you shall expresse it with your ryght hande, as here orderly by this table folowynge you may perceaue.

¶ Here foloweth the table of the arte of the hande

The arte of nombrynge by the hande.

page number '134' from original illustration

hand numbering as described in text

*In which as you may se 1 is expressed by y^e^ lyttle fynger of y^e^ lefte hande closely and harde croked.

declared by lyke bowynge of the weddynge fynger (whiche is the nexte to the lyttell fynger) together with the lytell fynger.

ignified by the myddle fynger bowed in lyke maner, with those other two.

declared by the bowyng of the myddle fynger and the rynge fynger, or weddynge fynger, with the other all stretched forth.

epresented by the myddle fynger onely bowed.

by the weddynge fynger only crooked: and this you may marke in these a certayne order. But now 7, 8, and 9, are expressed w*ith* the bowynge of the same fyngers as are 1, 2, and 3, but after an other fourme.

is declared by the bowynge of the lytell fynger, as is 1, saue that for 1 the fynger is clasped in, harde *and* 135 *a.* *rounde, but for to expresse 7, you shall bowe the myddle ioynte of the lytell fynger only, and holde the other ioyntes streyght.

S. Yf you wyll geue me leue to expresse it after my rude maner, thus I vnderstand your meanyng: that 1 is expressed by crookynge in the lyttell fynger lyke the head of a bysshoppes bagle: and 7 is declared by the same fynger bowed lyke a gybbet.

M. So I perceaue, you vnderstande it.

to expresse 8, you shall bowe after the same maner both the lyttell fynger and the rynge fynger.

/f you bowe lyke wayes with them the myddle fynger, then doth it betoken 9.

to expresse 10, you shall bowe your fore fynger rounde, and set the ende of it on the hyghest ioynte of the thombe.

for to expresse 20, you must set your fyngers streyght, and the ende of your thombe to the partition of the 135 *b.* *fore moste and myddle fynger.

represented by the ioynynge together of y^e^ headdes of the foremost fynger and the thombe.

declared by settynge of the thombe crossewayes on the foremost fynger.

signified by ryght stretchyng forth of the fyngers ioyntly, and applyenge of the thombes ende to the partition of the myddle fynger *and* the rynge fynger, or weddynge fynger.

formed by bendynge of the thombe croked and crossynge it with the fore fynger.

expressed by the bowynge of the foremost fynger, and settynge the ende of the thombe between the 2 foremost or hyghest ioyntes of it.

expressed by settynge of the foremost fynger crossewayes on the thombe, so that 80 dyffereth thus from 40, that for 80 the forefynger is set crosse on the thombe, and for 40 the thombe is set crosse ouer y^e^ forefinger.

*90 is signified, by bendynge the fore fynger, and settyng the ende of it 90
in the innermost ioynte of y^e thombe, that is euen at the foote of it. And thus are all the no*m*bers ended vnder 100.

S. In dede these be all the nombers fro*m* 1 to 10, *and* then all the tenthes within 100,
but this teacheth me not how to expresse 11, 12, 13, *et*c. 21, 22, 23, *et*c. 11, 12, 13, 21, 22, 23,
and such lyke.

M. You can lytell vnderstande, yf you can not do that without teachynge: what is 11? is it not 10 and 1? then expresse 10 as you were taught, and 1 also, and that is 11: and for 12 expresse 10 and 2: for 23 set 20 and 3: and so for 68 you muste make 60 and there to 8: and so of all other sortes.

now yf you wolde represente 100 other any nomber aboue it, you muste 100
do that with the ryghte hande, after this maner.

You must expresse 100 in the ryght hand, with the lytell fynger so bowed as you dyd expresse 1 in the left hand.

*And as you expressed 2 in the lefte hande, the same fasshyon in the 200
ryght hande doth declare 200.

fourme of 3 in the ryght hand standeth for 300. 300

fourme of 4, for 400. 400

ewayes the fourme of 5, for 500. 500

fourme of 6, for 600. And to be shorte: loke how you did expresse 600
single vnities and tenthes in the lefte hande, so must you expresse vnities *and* tenthes of hundredes, in the ryghte hande.

vnderstande you thus: that yf I wold represent 900, I must so fourme the 900
fyngers of my ryghte hande, as I shuld do in my left hand to expresse 9, and as in my lefte hand I expressed 10, so in my ryght hande must I expresse 1000.

And so the fourme of euery tenthe in the lefte hande serueth to expresse lyke no*m*ber of thousy*and*es, as the fourme of 40 standeth for 4000.

e fourme of 80 for 8000. 8000

a. 9000

*And the fourme of 90 (whiche is
the greatest) for 9000, and aboue that
I can not expresse any nomber. *M.*
No not with one fynger: how be it,
w*ith* dyuers fyngers you maye expresse
9999, and all at one tyme, and that lac
keth but 1 of 10000. So that vnder
10000 you may by your fyngers ex-
presse any summe. And this shal suf-
fyce for Numeration on the fyngers.
And as for Addition, Subtraction,
Multiplicatio*n*, and Diuision (which
yet were neuer taught by any man as
farre as I do knowe) I wyll enstruct
you after the treatyse of fractions.
And now for this tyme fare well,
and loke that you cease not to
practyse that you haue lear
ned. *S.* Syr, with moste
harty mynde I thanke
you, bothe for your
good learnyng, *and*
also your good
cou*ns*el, which
(god wyllyng) I truste to folow.

Finis.

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1. 1342 in original.
2. 168 in original.
3. Bracket ([) denotes new paragraph in original.

For this e-text, the brackets have been omitted in favor of restoring the paragraph breaks. Numbers 200 and up were printed as separate paragraphs and are unchanged. Sidenote 4 was missing and has been supplied by the

transcriber; the pairs 5, 6 and 9, 10 (originally on one line) have been separated.

APPENDIX I.

A Treatise on the Numeration of Algorism.

[*From a MS. of the 14th Century.*]

To alle suche even nombrys the most have cifrys as to ten. twenty. thirtty. an hundred. an thousand and suche other. but ye schal vnderstonde that a cifre tokeneth nothinge but he maketh other the more significatyf that comith after hym. Also ye schal vnderstonde that in nombrys composyt and in alle other nombrys that ben of diverse figurys ye schal begynne in the ritht syde and to rekene backwarde and so he schal be wryte as thus—1000. the sifre in the ritht side was first wryte and yit he tokeneth nothinge to the secunde no the thridde but thei maken that figure of 1 the more signyficatyf that comith after hem by as moche as he born oute of his first place where he schuld yf he stode ther tokene but one. And there he stondith nowe in the ferye place he tokeneth a thousand as by this rewle. In the first place he tokeneth but hymself. In the secunde place he tokeneth ten times hymself. In the thridde place he tokeneth an hundred tymes himself. In the ferye he tokeneth a thousand tymes himself. In the fyftye place he tokeneth ten thousand tymes himself. In the sexte place he tokeneth an hundred thousand tymes hymself. In the seveth place he tokeneth ten hundred thousand tymes hymself, &c. And ye schal vnderstond that this worde nombre is partyd into thre

partyes. Somme is callyd nombre of digitys for alle ben digitys that ben withine ten as ix, viii, vii, vi, v, iv, iii, ii, i. Articules ben alle thei that mow be devyded into nombrys of ten as xx, xxx, xl, and suche other. Compositтys be alle nombrys that ben componyd of a digyt and of an articule as fourtene fyftene thrittene and suche other. Fourtene is componyd of four that is a digyt and of ten that is an articule. Fyftene is componyd of fyve that is a digyt and of ten that is an articule and so of others . . . . . But as to this rewle. In the firste place he tokeneth but himself that is to say he tokeneth but that and no more. If that he stonde in the secunde place he tokeneth ten tymes himself as this figure 2 here 21. this is oon and twenty. This figure 2 stondith in the secunde place and therfor he tokeneth ten tymes himself and ten tymes 2 is twenty and so forye of every figure and he stonde after another toward the lest syde he schal tokene ten tymes as moche more as he schuld token and he stode in that place ther that the figure afore him stondeth: lo an example as thus 9634. This figure of foure that hath this schape 4 tokeneth but himself for he stondeth in the first place. The figure of thre that hath this schape 3 tokeneth ten tyme himself for he stondeth in the secunde place and that is thritti. The figure of sexe that hath this schape 6 tokeneth ten tyme more than he schuld and he stode in the place yer the figure of thre stondeth for ther he schuld tokene but sexty. And now he tokeneth ten tymes that is sexe hundrid. The figure of nyne that hath this schape 9 tokeneth ten tymes more than he schulde and he stode in the place ther the figure of 6 stondeth inne for thanne he schuld tokene but nyne hundryd. And in the place that he stondeth inne nowe he tokeneth nine thousand. Alle the hole nombre of these foure figurys. Nine thousand sexe hundrid and foure and thritti.

APPENDIX II.

Carmen de Algorismo.

[*From a B.M. MS.,* 8 *C.* iv., *with additions from* 12 *E.* 1 *& Eg.* 2622.]

Hec algorismus ars presens dicitur [1]; in qua
Talibus Indorum [2] fruimur his quinque figuris.
0. 9. 8. 7. 6. 5. 4. 3. 2. 1.
Prima significat unum: duo vero secunda:
4
Tercia significat tria: sic procede sinistre
Donec ad extremam venies, qua cifra vocatur;
[3][Que nil significat; dat significare sequenti.]
Quelibet illarum si primo limite ponas,
8
Simpliciter se significat: si vero secundo,
Se decies: sursum procedas multiplicando. [4]
[Namque figura sequens quevis signat decies plus,
12
Ipsa locata loco quam significet pereunte:
Nam precedentes plus ultima significabit.]

[5]Post predicta scias quod tres breuiter numerorum
Distincte species sunt; nam quidam digiti sunt;
16
Articuli quidam; quidam quoque compositi sunt.
[Sunt digiti numeri qui citra denarium sunt;
Articuli decupli degitorum; compositi sunt
Illi qui constant ex articulis digitisque.]
20
Ergo, proposito numero tibi scribere, primo
Respicias quis sit numerus; quia si digitus sit,
[5][Una figura satis sibi; sed si compositus sit,]

Primo scribe loco digitum post articulum fac
Articulus si sit, cifram post articulum sit,
[Articulum vero reliquenti in scribe figure.]
Quolibet in numero, si par sit prima figura,
Par erit et totum, quicquid sibi continetur;
Impar si fuerit, totum sibi fiet et impar.

Septem [6] sunt partes, non plures, istius artis;
Addere, subtrahere, duplare, dimidiare;
Sexta est diuidere, set quinta est multiplicare;
Radicem extrahere pars septima dicitur esse.
Subtrahis aut addis a dextris vel mediabis;
A leua dupla, diuide, multiplicaque;
Extrahe radicem semper sub parte sinistra.

Addere si numero numerum vis, ordine tali
Incipe; scribe duas primo series numerorum
Prima sub prima recte ponendo figuram,
Et sic de reliquis facias, si sint tibi plures.
Inde duas adde primas hac condicione;
Si digitus crescat ex addicione priorum,
Primo scribe loco digitum, quicunque sit ille;
Si sit compositus, in limite scribe sequenti
Articulum, primo digitum; quia sic iubet ordo.
Articulus si sit, in primo limite cifram,
Articulum vero reliquis inscribe figuris;
Vel per se scribas si nulla figura sequatur.
Si tibi cifra superueniens occurrerit, illam
Deme suppositam; post illic scribe figuram:
Postea procedas reliquas addendo figuras.

Subtraccion

A subtracto numerum si sit tibi demere cura,
Scribe figurarum series, vt in addicione;
Maiori numero numerum suppone minorem,
Siue pari numero supponatur numerus par.
Postea si possis a prima subtrahe primam,
Scribens quod remanet, cifram si nil remanebit.
Set si non possis a prima demere primam;
Procedens, vnum de limite deme sequenti;
Et demptum pro denario reputabis ab illo,
Subtrahe totaliter numerum quem proposuisti.
Quo facto, scribe supra quicquit remanebit,
Facque novenarios de cifris, cum remanebis,
Occurrant si forte cifre, dum demseris vnum;
Postea procedas reliquas demendo figuras.

Proba

[7][Si subtraccio sit bene facta probare valebis,
Quas subtraxisti primas addendo figuras.
Nam, subtractio si bene sit, primas retinebis,
Et subtractio facta tibi probat additionem.]

Duplum

Si vis duplare numerum, sic incipe; solam
Scribe figurarum seriem, quamcumque voles que
Postea procedas primam duplando figuram;
Inde quod excrescet, scribens, vbi iusserit ordo,
Juxta precepta que dantur in addicione.
Nam si sit digitus, in primo limite scribe;
Articulus si sit, in primo limite cifram,
Articulum vero reliquis inscribe figuris;
Vel per se scribas, si nulla figura sequatur:
Compositus si sit, in limite scribe sequenti

Articulum primo, digitum; quia sic jubet ordo:
Et sic de reliquis facias, si sint tibi plures.
[8][Si super extremam nota sit, monadem dat eidem,
Quod tibi contingit, si primo dimidiabis.]

Incipit sic. Mediatio. Vpe sic, si vis aliquem numerum mediare:
Scribe figurarum seriem solam, velud ante;
Postea procedens medias, et prima figura
Si par aut impar videas; quia si fuerit par,
Dimidiabis eam, scribens quicquit remanebit;
Impar si fuerit, vnum demas, mediare,
Nonne presumas, sed quod superest mediabis;
Inde super tractum, fac demptum quod notat unum;
Si monos, dele; sit ibi cifra post nota supra.
Postea procedas hac condicione secunda: [9]
Impar [10] si fuerit hic vnum deme priori,
Inscribens quinque, nam denos significabit
Monos prædictam: si vero secunda dat vnam,
Illa deleta, scribatur cifra; priori
Tradendo quinque pro denario mediato;
Nec cifra scribatur, nisi inde figura sequatur:
Postea procedas reliquas mediando figuras,
Quin supra docui, si sint tibi mille figure.
[11][Si mediatio sit bene facta probare valebis,
Duplando numerum quem primo dimidiasti.]

Multiplicatio. Si tu per numerum numerum vis multiplicare,
Scribe duas, quascunque volis, series numerorum;
Ordo tamen seruetur vt vltima multiplicandi
Ponatur super anteriorem multiplicantis;

[12][A leua relique sint scripte multiplicantes.]
In digitum cures digitum si ducere, major
Per quantes distat a denis respice, debes
Namque suo decuplo tociens delere minorem;
Sicque tibi numerus veniens exinde patebit.
Postea procedas postremam multiplicando,
Juste multiplicans per cunctas inferiores,
Condicione tamen tali; quod multiplicantis
Scribas in capite, quicquid processerit inde;
Set postquam fuerit hec multiplicata, figure
Anteriorentur seriei multiplicantis;
Et sic multiplica, velut istam multiplicasti,
Qui sequitur numerum scriptum quicunque figuris.
Set cum multiplicas, primo sic est operandum,
Si dabit articulum tibi multiplicacio solum;
Proposita cifra, summam transferre memento.
Sin autem digitus excrescerit articulusque,
Articulus supraposito digito salit ultra;
Si digitus tamen, ponas illum super ipsam,
Subdita multiplicans hanc que super incidit illi
Delet eam penitus, scribens quod provenit inde;
Sed si multiplices illam posite super ipsam,
Adiungens numerum quem prebet ductus earum;
Si supraimpositam cifra debet multiplicare,
Prorsus eam delet, scribi que loco cifra debet,
[12][Si cifra multiplicat aliam positam super ipsam,
Sitque locus supra vacuus super hanc cifra fiet;]
Si supra fuerit cifra semper pretereunda est;
Si dubites, an sit bene multiplicando secunda,

136
Diuide totalem numerum per multiplicantem,
Et reddet numerus emergens inde priorem.

Mental Multiplication.

[13][Per numerum si vis numerum quoque multiplicare
Tantum per normas subtiles absque figuris
140
Has normas poteris per versus scire sequentes.
Si tu per digitum digitum quilibet multiplicabis
Regula precedens dat qualiter est operandum
Articulum si per reliquum vis multiplicare
144
In proprium digitum debebit uterque resolvi
Articulus digitos post per se multiplicantes
Ex digitis quociens teneret multiplicatum
Articuli faciunt tot centum multiplicati.
148
Articulum digito si multiplicamus oportet
Articulum digitum sumi quo multiplicare
Debemus reliquum quod multiplicaris ab illis
Per reliquo decuplum sic omne latere nequibit
152
In numerum mixtum digitum si ducere cures
Articulus mixti sumatur deinde resolvas
In digitum post hec fac ita de digitis nec
Articulusque docet excrescens in detinendo
156
In digitum mixti post ducas multiplicantem
De digitis ut norma docet sit juncta secundo
Multiplica summam et postea summa patebit
Junctus in articulum purum articulumque
160
[14][Articulum purum comittes articulum que]
Mixti pro digitis post fiat et articulus vt
Norma jubet retinendo quod egreditur ab illis
Articuli digitum post in digitum mixti duc

Regula de digitis ut percipit articulusque
Ex quibus excrescens summe tu junge priori
Sic manifesta cito fiet tibi summa petita.
Compositum numerum mixto sic multiplicabis
Vndecies tredecem sic est ex hiis operandum
In reliquum primum demum duc post in eundem
Unum post deinde duc in tercia deinde per unum
Multiplices tercia demum tunc omnia multiplicata
In summa duces quam que fuerit te dices
Hic ut hic mixtus intentus est operandum
Multiplicandorum de normis sufficiunt hec.]

Division.

Si vis dividere numerum, sic incipe primo;
Scribe duas, quascunque voles, series numerorum;
Majori numero numerum suppone minorem,
[15][Nam docet ut major teneat bis terve minorem;]
Et sub supprima supprimam pone figuram,
Sic reliquis reliquas a dextra parte locabis;
Postea de prima primam sub parte sinistra
Subtrahe, si possis, quociens potes adminus istud,
Scribens quod remanet sub tali conditione;
Ut totiens demas demendas a remanente,
Que serie recte ponentur in anteriori,
Unica si, tantum sit ibi decet operari;
Set si non possis a prima demere primam,
Procedas, et eam numero suppone sequenti;
Hanc uno retrahendo gradu quo comites retrahantur,
Et, quotiens poteris, ab eadem deme priorem,
Ut totiens demas demendas a remanenti,

Nec plus quam novies quicquam tibi demere debes,
Nascitur hinc numerus quociens supraque sequentem
Hunc primo scribas, retrahas exinde figuras,
Dum fuerit major supra positus inferiori,
Et rursum fiat divisio more priori;
Et numerum quotiens supra scribas pereunti,
Si fiat saliens retrahendo, cifra locetur,
Et pereat numero quotiens, proponas eidem
Cifram, ne numerum pereat vis, dum locus illic
Restat, et expletis divisio non valet ultra:
Dum fuerit numerus numerorum inferiore seorsum
Illum servabis; hinc multiplicando probabis,

Proof.

Si bene fecisti, divisor multiplicetur
Per numerum quotiens; cum multiplicaveris, adde
Totali summæ, quod servatum fuit ante,
Reddeturque tibi numerus quem proposuisti;
Et si nil remanet, hunc multiplicando reddet,

Square Numbers.

Cum ducis numerum per se, qui provenit inde
Sit tibi quadratus, ductus radix erit hujus,
Nec numeros omnes quadratos dicere debes,
Est autem omnis numerus radix alicujus.
Quando voles numeri radicem querere, scribi
Debet; inde notes si sit locus ulterius impar,
Estque figura loco talis scribenda sub illo,
Que, per se dicta, numerum tibi destruat illum,
Vel quantum poterit ex inde delebis eandem;
Vel retrahendo duples retrahens duplando sub ista
Que primo sequitur, duplicatur per duplacationem,

Post per se minuens pro posse quod est minuendum.
[16]Post his propones digitum, qui, more priori
Per precedentes, post per se multiplicatus,
Destruat in quantum poterit numerum remanentem,
Et sic procedens retrahens duplando figuram,
Preponendo novam donec totum peragatur,
Subdupla propriis servare docetque duplatis;
Si det compositum numerum duplacio, debet
Inscribi digitus a parte dextra parte propinqua,
Articulusque loco quo non duplicata resessit;
Si dabit articulum, sit cifra loco pereunte
Articulusque locum tenet unum, de duplicata resessit;
Si donet digitum, sub prima pone sequente,
Si supraposita fuerit duplicata figura
Major proponi debet tantummodo cifra,
Has retrahens solito propones more figuram,
Usque sub extrema ita fac retrahendo figuras,
Si totum deles numerum quem proposuisti,
Quadratus fuerit, de dupla quod duplicasti,
Sicque tibi radix illius certa patebit,
Si de duplatis fit juncta supprima figura;
Radicem per se multiplices habeasque
Primo propositum, bene te fecisse probasti;
Non est quadratus, si quis restat, sed habentur
Radix quadrati qui stat major sub eadem;
Vel quicquid remanet tabula servare memento;
Hoc casu radix per se quoque multiplicetur,
Vel sic quadratus sub primo major habetur,

Hinc addas remanens, et prius debes haberi;
Si locus extremus fuerit par, scribe figuram
Sub pereunte loco per quam debes operari,
Que quantum poterit supprimas destruat ambas,
Vel penitus legem teneas operando priorem,
Si suppositum digitus suo fine repertus,
Omnino delet illic scribi cifra debet,
A leva si qua sit ei sociata figura;
Si cifre remanent in fine pares decet harum
Radices, numero mediam proponere partem,
Tali quesita radix patet arte reperta.
Per numerum recte si nosti multiplicare
Ejus quadratum, numerus qui pervenit inde
Dicetur cubicus; primus radix erit ejus;
Nec numeros omnes cubicatos dicere debes,
Est autem omnis numerus radix alicujus;

Cube Root.

Si curas cubici radicem quærere, primo
Inscriptum numerum distinguere per loca debes;
Que tibi mille notant a mille notante suprema
Initiam, summa operandi parte sinistra,
Illic sub scribas digitum, qui multiplicatus
In semet cubice suprapositum sibi perdat,
Et si quid fuerit adjunctum parte sinistra
Si non omnino, quantum poteris minuendo,
Hinc triplans retrahe saltum, faciendo sub illa
Que manet a digito deleto terna, figuram
Illi propones quo sub triplo asocietur,
Ut cum subtriplo per eam tripla multiplicatur;
Hinc per eam solam productum multiplicabis,

Postea totalem numerum, qui provenit inde
A suprapositis respectu tolle triplate
Addita supprimo cubice tunc multiplicetur,
Respectu cujus, numerus qui progredietur
Ex cubito ductu, supra omnes adimetur;
Tunc ipsam delens triples saltum faciendo,
Semper sub ternas, retrahens alias triplicatas
Ex hinc triplatis aliam propone figuram,
Que per triplatas ducatur more priori;
Primo sub triplis sibi junctis, postea per se,
In numerum ducta, productum de triplicatis:
Utque prius dixi numerus qui provenit inde
A suprapositis has respiciendo trahatur,
Huic cubice ductum sub primo multiplicabis,
Respectumque sui, removebis de remanenti,
Et sic procedas retrahendo triplando figuram.
Et proponendo nonam, donec totum peragatur,
Subtripla sub propriis servare decet triplicatis;
Si nil in fine remanet, numerus datus ante
Est cubicus; cubicam radicem sub tripla prebent,
Cum digito juncto quem supprimo posuisti,
Hec cubice ducta, numerum reddant tibi primum.
Si quid erit remanens non est cubicus, sed habetur
Major sub primo qui stat radix cubicam,
Servari debet quicquid radice remansit,
Extracto numero, decet hec addi cubicato.
Quo facto, numerus reddi debet tibi primus.
Nam debes per se radicem multiplicare
Ex hinc in numerum duces, qui provenit inde

Sub primo cubicus major sic invenietur;
Illi jungatur remanens, et primus habetur,
Si per triplatum numerum nequeas operari;
Cifram propones, nil vero per hanc operare
Set retrahens illam cum saltu deinde triplata,
Propones illi digitum sub lege priori,
Cumque cifram retrahas saliendo, non triplicabis,
Namque nihil cifre triplacio dicitur esse;
At tu cum cifram protraxeris aut triplicata,
Hanc cum subtriplo semper servare memento:
Si det compositum, digiti triplacio debet
Illius scribi, digitus saliendo sub ipsam;
Digito deleto, que terna dicitur esse;
Jungitur articulus cum triplata pereunte,
Set facit hunc scribi per se triplacio prima,
Que si det digitum per se scribi facit illum;
Consumpto numero, si sole fuit tibi cifre
Triplato, propone cifram saltum faciendo,
Cumque cifram retrahe triplam, scribendo figuram,
Preponas cifre, sic procedens operare,
Si tres vel duo serie in sint, pone sub yma,
A dextris digitum servando prius documentum.
Si sit continua progressio terminus nuper
Per majus medium totalem multiplicato;
Si par, per medium tunc multiplicato sequentem.
Set si continua non sit progressio finis:
Impar, tunc majus medium si multiplicabis,
Si par per medium sibi multiplicato propinquum.

1. “Hec præsens ars dicitur algorismus ab Algore rege ejus inventore, vel dicitur ab *algos* quod est ars, et *rodos* quod est numerus; quæ est ars numerorum vel numerandi, ad quam artem bene sciendum inveniebantur apud Indos bis quinque (id est decem) figuræ.” —*Comment. Thomæ de Novo-Mercatu.* MS. Bib. Reg. Mus. Brit. 12 E. 1.

2. “Hæ necessariæ figuræ sunt Indorum characteros.” *MS. de numeratione.* Bib. Sloan. Mus. Brit. 513, fol. 58. “Cum vidissem Yndos constituisse IX literas in universo numero suo propter dispositionem suam quam posuerunt, volui patefacere de opere quod sit per eas aliquidque esset levius discentibus, si Deus voluerit. Si autem Indi hoc voluerunt et intentio illorum nihil novem literis fuit, causa que mihi potuit. Deus direxit me ad hoc. Si vero alia dicam preter eam quam ego exposui, hoc fecerunt per hoc quod ego exposui, eadem tam certissime et absque ulla dubitatione poterit inveniri. Levitasque patebit aspicientibus et discentibus.” MS. U.L.C., Ii. vi. 5, f. 102.

3. From Eg. 2622.

4. 8 C. iv. inserts Nullum cipa significat: dat significare sequenti.

5. From 12 E. 1.

6.

> En argorisme devon prendre
> Vii especes . . . .
> Adision subtracion
> Doubloison mediacion
> Monteploie et division
> Et de radix eustracion
> A chez vii especes savoir

Doit chascun en memoire avoir
Letres qui figures sont dites
Et qui excellens sont ecrites.—MS. *Seld. Arch.* B. 26.

7. From 12 E. 1.

8. From 12 E. 1.

9. 8 C. iv. inserts Atque figura prior nuper fuerit mediando.

10. *I.e.* figura secundo loco posita.

11. So 12 E. 1; 8 C. iv. inserts—

Si super extremam nota sit monades dat eidem
Quod contingat cum primo dimiabis
Atque figura prior nuper fuerit mediando.

12. 12 E. 1 inserts.

13. 12 E. 1 inserts to l. 174.

14. 12 E. 1 omits, Eg. 2622 inserts.

15. 12 E. 1 inserts.

16. 8 C. iv. inserts—

Hinc illam dele duplans sub ei psalliendo
Que sequitur retrahens quicquid fuerit duplicatum.

INDEX OF TECHNICAL TERMS [1]

algorisme, 33/12; **algorym**, **augrym**, 3/3; the art of computing, using the so-called Arabic numerals.

The word in its various forms is derived from the Arabic *al-Khowarazmi* (i.e. the native of Khwarazm (Khiva)). This was the surname of Ja'far Mohammad ben Musa, who wrote a treatise early in the 9th century (see p. xiv).

The form *algorithm* is also found, being suggested by a supposed derivation from the Greek ἀριθμός (number).

antery, 24/11; to move figures to the right of the position in which they are first written. This operation is performed repeatedly upon the multiplier in multiplication, and upon certain figures which arise in the process of root extraction.

anterioracioun, 50/5; the operation of moving figures to the right.

article, 34/23; **articul**, 5/31; **articuls**, 9/36, 29/7,8; a number divisible by ten without remainder.

cast, 8/12; to add one number to another.

'Addition is a *casting* together of two numbers into one number,' 8/10.

cifre, 4/1; the name of the figure 0. The word is derived from the Arabic *sifr* = empty, nothing. Hence *zero*.

A cipher is the symbol of the absence of number or of zero quantity. It may be used alone or in conjunction with digits or other ciphers, and in the latter case, according to the position which it occupies relative to the other figures, indicates the absence of units, or tens, or hundreds, etc. The great superiority of the Arabic to all other systems of notation resides in the employment of this symbol. When the cipher is not used, the place value of digits has to be indicated by writing them in assigned rows or columns. Ciphers, however, may be interpolated amongst the significant figures used, and as they sufficiently indicate the positions of the empty rows or columns, the latter need not be indicated in any other way. The practical performance of calculations is thus enormously facilitated (see p. xvi).

componede, 33/24; **composyt**, 5/35; with reference to numbers, one compounded of a multiple of ten and a digit.

conuertide = conversely, 46/29, 47/9.

cubicede, 50/13; **to be c.**, to have its cube root found.

cubike nombre, 47/8; a number formed by multiplying a given number twice by itself, *e.g.* 27 = 3 × 3 × 3. Now called simply a cube.

decuple, 22/12; the product of a number by ten. Tenfold.

departys = divides, 5/29.

digit, 5/30; **digitalle**, 33/24; a number less than ten, represented by one of the nine Arabic numerals.

dimydicion, 7/23; the operation of dividing a number by two. Halving.

duccioun, multiplication, 43/9.

duplacion, 7/23, 14/15; the operation of multiplying a number by two. Doubling.

i-mediet = halved, 19/23.

intercise = broken, 46/2; intercise Progression is the name given to either of the Progressions 1, 3, 5, 7, etc.; 2, 4, 6, 8, etc., in which the common difference is 2.

lede into, multiply by, 47/18.

lyneal nombre, 46/14; a number such as that which expresses the measure of the length of a line, and therefore is not *necessarily* the product of two or more numbers (*vide* Superficial, Solid). This appears to be the meaning of the phrase as used in *The Art of Nombryng*. It is possible that the numbers so designated are the prime numbers, that is, numbers not divisible by any other number except themselves and unity, but it is not clear that this limitation is intended.

mediacioun, 16/36, 38/16; dividing by two (see also **dimydicion**).

medlede nombre, 34/1; a number formed of a multiple of ten and a digit (*vide* componede, composyt).

medye, 17/8, to halve; **mediete**, halved, 17/30; **ymedit**, 20/9.

naturelle progressioun, 45/22; the series of numbers 1, 2, 3, etc.

produccioun, multiplication, 50/11.

quadrat nombre, 46/12; a number formed by multiplying a given number by itself, *e.g.* $9 = 3 \times 3$, a square.

rote, 7/25; **roote**, 47/11; root. The roots of squares and cubes are the numbers from which the squares and cubes are derived by multiplication into themselves.

significatyf, significant, 5/14; The significant figures of a number are, strictly speaking, those other than zero, *e.g.* in 3 6 5 0 4 0 0, the significant figures are 3, 6, 5, 4. Modern usage, however, regards all figures between the two extreme significant figures as significant, even when some are zero. Thus, in the above example, 3 6 5 0 4 are considered significant.

solide nombre, 46/37; a number which is the product of three other numbers, *e.g.* $66 = 11 \times 2 \times 3$.

superficial nombre, 46/18; a number which is the product of two other numbers, *e.g.* $6 = 2 \times 3$.

ternary, consisting of three digits, 51/7.

vnder double, a digit which has been doubled, 48/3.

vnder-trebille, a digit which has been trebled, 49/28; **vnder-triplat**, 49/39.

w, a symbol used to denote half a unit, 17/33.

1. This Index has been kindly prepared by Professor J. B. Dale, of King's College, University of London, and the best thanks of the Society are due to him for his valuable contribution.

GLOSSARY

Words whose first appearance is earlier than the page cited in the Glossary are identified in supplementary notes, and both occurrences are marked in the

main text.

ablacioun, taking away, 36/21
addyst, haddest, 10/37
agregacioun, addition, 45/22. (First example in N.E.D., 1547.)
a-ȝenenes, against, 23/10
allgate, always, 8/39
als, as, 22/24
and, if, 29/8;
    &, 4/27;
    & yf, 20/7
a-nendes, towards, 23/15
aproprede, appropriated, 34/27
apwereth, appears, 61/8
a-risyȝt, arises, 14/24
a-rowe, in a row, 29/10
arsemetrike, arithmetic, 33/1
ayene, again, 45/15
bagle, crozier, 67/12
bordure = ordure, row, 43/30
borro, *inf.* borrow, 11/38;
    imp. s. **borowe**, 12/20;
    pp. **borwed**, 12/15;
    borred, 12/19
boue, above, 42/34
caputule, chapter, 7/26
certayn, assuredly, 18/34
clepede, called, 47/7
competently, conveniently, 35/8
compt, count, 47/29
contynes, contains, 21/12;
    pp. **contenythe**, 38/39
craft, art, 3/4
distingue, divide, 51/5
egalle, equal, 45/21
excep, except, 5/16
exclusede, excluded, 34/37
excressent, resulting, 35/16
exeant, resulting, 43/26

expone, expound, 3/23
ferye = ferþe, fourth, 70/12
figure = figures, 5/1
for-by, past, 12/11
fors; no f., no matter, 22/24
forseth, matters, 53/30
forye = forþe, forth, 71/8
fyftye = fyftþe, fifth, 70/16
grewe, Greek, 33/13
haluendel, half, 16/16;
    haldel, 19/4;
    pl. **haluedels**, 16/16
hayst, hast, 17/3, 32
hast, haste, 22/25
heer, higher, 9/35
here, their, 7/26
here-a-fore, heretofore, 13/7
heyth, was called, 3/5
hole, whole, 4/39;
    holle, 17/1;
    hoole, of three dimensions, 46/15
holdyþe, holds good, 30/5
how be it that, although, 44/4
lede = lete, let, 8/37
lene, lend, 12/39
lest, least, 43/27
lest = left, 71/9
leue, leave, 6/5;
    pr. 3 s. **leues**, remains, 11/19;
    leus, 11/28;
    pp. **laft**, left, 19/24
lewder, more ignorant, 3/3
lust, desirest to, 45/13
lyȝt, easy, 15/31
lymytes, limits, 34/18;
    lynes, 34/12;
    lynees, 34/17;
    Lat. limes, *pl.* limites.
maystery, achievement;

First used in 10/40

no m., no achievement, i.e. easy, 19/10

me, *indef. pron.* one, 42/1 First used in 34/16

mo, more, 9/16

moder = more (Lat. majorem), 43/22

most, must, 30/3 First used in 3/12

multipliede, **to be m.** = multiplying, 40/9

mynvtes, the sixty parts into which a unit is divided, 38/25

myse-wroȝt, mis-wrought, 14/11

nether, nor, 34/25

nex, next, 19/9

noȝt, nought, 5/7 First used in 4/8

note, not, 30/5

oo, one, 42/20; **o**, 42/21 First used in 34/27 (oo); 33/22 (o)

omest, uppermost, higher, 35/26;

omyst, 35/28

omwhile, sometimes, 45/31 First used in 39/17

on, one, 8/29

opyne, plain, 47/8

or, before, 13/25

or = þe oþ*er*, the other, 28/34

ordure, order, 34/9;

row, 43/1 Word form is "order"

other, or, 33/13, 43/26; Note also "one other other" in 35/24

other . . . or, either . . . or, 38/37 First used in 37/5

ouerer, upper, 42/15

ouer-hippede, passed over, 43/19

recte, directly, 27/20 First used in 26/31

remayner, remainder, 56/28

representithe, represented, 39/14

resteth, remains, 63/29 First used in 57/29

rewarde, regard, 48/6

rew, row, 4/8

rewle, row, 4/20, 7/12;

rewele, 4/18;

rewles, rules, 5/33

s. = scilicet, 3/8

sentens, meaning, 14/29

signifye(tyf), 5/13. The last three letters are added above the line, evidently because of the word 'significatyf' in l. 14. But the 'Solucio,' which contained the word, has been omitted.

sithen, since, 33/8

some, sum, result, 40/17, 32 First used in 36/21
sowne, pronounce, 6/29
singillatim, singly, 7/25
spices, species, kinds, 34/4 First used in 5/34
spyl, waste, 14/26
styde, stead, 18/20
subtrahe, subtract, 48/12;
pp. **subtrayd**, 13/21
sythes, times, 21/16
taȝt, taught, 16/36
take, *pp.* taken;
t. fro, starting from, 45/22
taward, toward, 23/34
thouȝt, though, 5/20
trebille, multiply by three, 49/26
twene, two, 8/11 First used in 4/23
þow, though, 25/15
þowȝt, thought;
be þ., mentally, 28/4
þus = þis, this, 20/33
vny, unite, 45/10
wel, wilt, 14/31
wete, wit, 15/16;
wyte, know, 8/38;
pr. 2 s. **wost**, 12/38
wex, become, 50/18
where, whether, 29/12
wher-thurghe, whence, 49/15
worch, work, 8/19; First used in 7/35
wrich, 8/35;
wyrch, 6/19;
imp. s. **worch**, 15/9; First used in 9/6
pp. **y-wroth**, 13/24
write, written, 29/19; First used in 4/5
y-write, 16/1
wryrchynge = wyrchynge, working, 30/4
wt, with, 55/8
y-broth, brought, 21/18
ychon, each one, 29/10

ydo, done, added, 9/6 First used in 8/37

ylke, same, 5/12

y-lyech, alike, 22/23

y-my3t, been able, 12/2

y-now3t, enough, 15/31;

ynov3t, 18/34

yove, given, 45/33

t

y , that, 52/8

y-write, *v.* **write.**

y-wroth, *v.* **worch.**

MARGINAL NOTES

Headnotes have been moved to the beginning of the appropriate paragraph. Headnotes were omitted from the two Appendixes, as sidenotes give the same information.

Line Numbers are cited in the Index and Glossary. They have been omitted from the e-text except in the one verse selection (App. II, *Carmen de Algorismo*). Instead, the Index and Glossary are linked directly to each word.

Numbered Notes:

Numbered sidenotes show page or leaf numbers from the original MSS. In the e-text, sidenote numbers have been replaced with simple asterisks.

Footnotes give textual information such as variant readings. They have been numbered sequentially within each title.

Sidenotes giving a running synopsis of the text have been kept as close as possible to their original format and location.

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