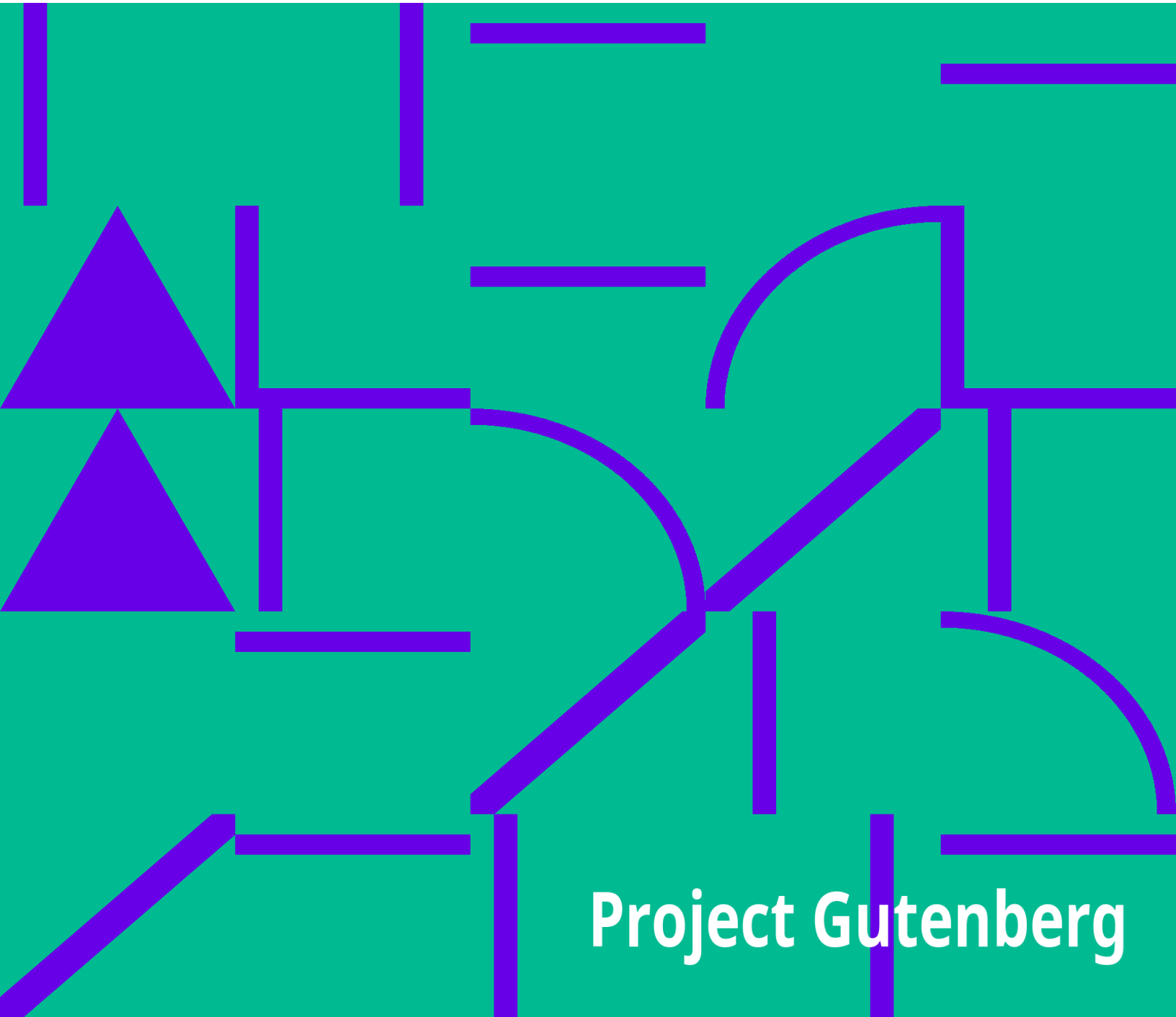


The calculating engine

Charles Babbage



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*** START OF THE PROJECT GUTENBERG EBOOK THE
CALCULATING ENGINE ***

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THE CALCULATING ENGINE

BY

CHARLES BABBAGE

- Art I.—1. *Letter to Sir Humphry Davy, Bart. P.R.S., on the application of Machinery to Calculate and Print Mathematical Tables.* By CHARLES BABBAGE, Esq. F.R.S. 4to. Printed by order of the House of Commons.
2. *On the Application of Machinery to the Calculation of Astronomical and Mathematical Tables.* By CHARLES BABBAGE, Esq. Memoirs Astron. Soc. Vol. I. Part 2. London: 1822.
3. *Address to the Astronomical Society, by Henry Thomas Colebrooke, Esq. F.R.S. President, on presenting the first gold medal of the Society to Charles Babbage, Esq. for the invention of the Calculating Engine.* Memoirs Astron. Soc. Vol. I. Part 2. London: 1822.
4. *On the determination of the General Term of a new Class of Infinite Series.* By CHARLES BABBAGE, Esq. Transactions Camb. Phil. Soc. Cambridge: 1824.
5. *On Errors common to many Tables of Logarithms.* By CHARLES BABBAGE, Esq. Memoirs Astron. Soc. London: 1827.
6. *On a Method of Expressing by Signs the Action of Machinery.* By CHARLES BABBAGE, Esq. Phil. Trans. London: 1826.
7. *Report by the Committee appointed by the Council of the Royal Society to consider the subject referred to in a Communication received by them from the Treasury, respecting Mr Babbage's Calculating Engine, and to report thereupon.* London: 1829.

THESE is no position in society more enviable than that of the few who unite a moderate independence with high intellectual qualities. Liberated from the necessity of seeking their support by a profession, they are unfettered by its restraints, and are enabled to direct the powers of their

minds, and to concentrate their intellectual energies on those objects exclusively to which they feel that their powers may be applied with the greatest advantage to the community, and with the most lasting reputation to themselves. On the other hand, their middle station and limited income rescue them from those allurements to frivolity and dissipation, to which rank and wealth ever expose their possessors. Placed in such favourable circumstances, Mr Babbage selected science as the field of his ambition; and his mathematical researches have conferred on him a high reputation, wherever the exact sciences are studied and appreciated. The suffrages of the mathematical world have been ratified in his own country, where he has been elected to the Lucasian Professorship in his own University—a chair, which, though of inconsiderable emolument, is one on which Newton has conferred everlasting celebrity. But it has been the fortune of this mathematician to surround himself with fame of another and more popular kind, and which rarely falls to the lot of those who devote their lives to the cultivation of the abstract sciences. This distinction he owes to the announcement, some years since, of his celebrated project of a Calculating Engine. A proposition to reduce arithmetic to the dominion of mechanism,—to substitute an automaton for a compositor,—to throw the powers of thought into wheel-work could not fail to awaken the attention of the world. To bring the practicability of such a project within the compass of popular belief was not easy: to do so by bringing it within the compass of popular comprehension was not possible. It transcended the imagination of the public in general to conceive its possibility; and the sentiments of wonder with which it was received, were only prevented from merging into those of incredulity, by the faith reposed in the high attainments of its projector. This extraordinary undertaking was, however, viewed in a very different light by the small section of the community, who, being sufficiently versed in mathematics, were acquainted with the principle upon which it was founded. By reference to that principle, they perceived at a glance the practicability of the project; and being enabled by the nature of their attainments and pursuits to appreciate the immeasurable importance of its results, they regarded the invention with a proportionately profound interest. The production of numerical tables, unlimited in quantity and variety, restricted to no particular species, and limited by no particular law;—extending not merely to the boundaries of existing knowledge, but spreading their powers over the undefined regions of future discovery—

were results, the magnitude and the value of which the community in general could neither comprehend nor appreciate. In such a case, the judgment of the world could only rest upon the authority of the philosophical part of it; and the fiat of the scientific community swayed for once political councils. The British Government, advised by the Royal Society, and a committee formed of the most eminent mechanics and practical engineers, determined on constructing the projected mechanism at the expense of the nation, to be held as national property.

Notwithstanding the interest with which this invention has been regarded in every part of the world, it has never yet been embodied in a written, much less in a published form. We trust, therefore, that some credit will be conceded to us for having been the first to make the public acquainted with the object, principle, and structure of a piece of machinery, which, though at present unknown (except as to a few of its probable results), must, when completed, produce important effects, not only on the progress of science, but on that of civilisation.

The calculating machinery thus undertaken for the public gratuitously (so far as Mr Babbage is concerned), has now attained a very advanced stage towards completion; and a portion of it has been put together, and performs various calculations;—affording a practical demonstration that the anticipations of those, under whose advice Government has acted, have been well founded.

There are nevertheless many persons who, admitting the great ingenuity of the contrivance, have, notwithstanding, been accustomed to regard it more in the light of a philosophical curiosity, than an instrument for purposes practically useful. This mistake (than which it is not possible to imagine a greater) has arisen mainly from the ignorance which prevails of the extensive utility of those numerical tables which it is the purpose of the engine in question to produce. There are also some persons who, not considering the time requisite to bring any invention of this magnitude to perfection in all its details, incline to consider the delays which have taken place in its progress as presumptions against its practicability. These persons should, however, before they arrive at such a conclusion, reflect upon the time which was necessary to bring to perfection engines infinitely inferior in complexity and mechanical difficulty. Let them remember that—

not to mention the *invention* of that machine—the *improvements* alone introduced into the steam-engine by the celebrated Watt, occupied a period of not less than twenty years of the life of that distinguished person, and involved an expenditure of capital amounting to L.50,000.^[1] The calculating machinery is a contrivance new even in its details. Its inventor did not take it up already imperfectly formed, after having received the contributions of human ingenuity exercised upon it for a century or more. It has not, like almost all other great mechanical inventions, been gradually advanced to its present state through a series of failures, through difficulties encountered and overcome by a succession of projectors. It is not an object on which the light of various minds has thus been shed. It is, on the contrary, the production of solitary and individual thought,—begun, advanced through each successive stage of improvement, and brought to perfection by one mind. Yet this creation of genius, from its first rude conception to its present state, has cost little more than half the time, and not one-third of the expense, consumed in bringing the steam-engine (previously far advanced in the course of improvement) to that state of comparative perfection in which it was left by Watt. Short as the period of time has been which the inventor has devoted to this enterprise, it has, nevertheless, been demonstrated, to the satisfaction of many scientific men of the first eminence, that the design in all its details, reduced, as it is, to a system of mechanical drawings, is complete; and requires only to be constructed in conformity with those plans, to realize all that its inventor has promised.

[1]Watt commenced his investigations respecting the steam-engine in 1763, between which time, and the year 1782 inclusive, he took out several patents for improvements in details. Bolton and Watt had expended the above sum on their improvements before they began to receive any return.

With a view to remove and correct erroneous impressions, and at the same time to convert the vague sense of wonder at what seems incomprehensible, with which this project is contemplated by the public in general, into a more rational and edifying sentiment, it is our purpose in the present article.

First, To show, the immense importance of any method by which numerical tables, absolutely accurate in every individual copy, may be produced with facility and cheapness. This we shall establish by conveying to the reader some notion of the number and variety of tables published in

every country of the world to which civilisation has extended, a large portion of which have been produced at the public expense; by showing also, that they are nevertheless rendered inefficient, to a greater or less extent, by the prevalence of errors in them; that these errors pervade not merely tables produced by individual labour and enterprise, but that they vitiate even those on which national resources have been prodigally expended, and to which the highest mathematical ability, which the most enlightened nations of the world could command, has been unsparingly and systematically directed.

Secondly, To attempt to convey to the reader a general notion of the mathematical principle on which the calculating machinery is founded, and of the manner in which this principle is brought into practical operation, both in the process of calculating and printing. It would be incompatible with the nature of this review, and indeed impossible without the aid of numerous plans, sections, and elevations, to convey clear and precise notions of the details of the means by which the process of reasoning is performed by inanimate matter, and the arbitrary and capricious evolutions of the fingers of typographical compositors are reduced to a system of wheel-work. We are, nevertheless, not without hopes of conveying, even to readers unskilled in mathematics, some satisfactory notions of a general nature on this subject.

Thirdly, To explain the actual state of the machinery at the present time; what progress has been made towards its completion; and what are the probable causes of those delays in its progress, which must be a subject of regret to all friends of science. We shall indicate what appears to us the best and most practicable course to prevent the unnecessary recurrence of such obstructions for the future, and to bring this noble project to a speedy and successful issue.

Viewing the infinite extent and variety of the tables which have been calculated and printed, from the earliest periods of human civilisation to the present time, we feel embarrassed with the difficulties of the task which we have imposed on ourselves;—that of attempting to convey to readers unaccustomed to such speculations, any thing approaching to an adequate idea of them. These tables are connected with the various sciences, with

almost every department of the useful arts, with commerce in all its relations; but above all, with Astronomy and Navigation. So important have they been considered, that in many instances large sums have been appropriated by the most enlightened nations in the production of them; and yet so numerous and insurmountable have been the difficulties attending the attainment of this end, that after all, even navigators, putting aside every other department of art and science, have, until very recently, been scantily and imperfectly supplied with the tables indispensably necessary to determine their position at sea.

The first class of tables which naturally present themselves, are those of Multiplication. A great variety of extensive multiplication tables have been published from an early period in different countries; and especially tables of *Powers*, in which a number is multiplied by itself successively. In Dodson's *Calculator* we find a table of multiplication extending as far as 10 times 1000.^[2] In 1775, a still more extensive table was published to 10 times 10,000. The Board of Longitude subsequently employed the late Dr Hutton to calculate and print various numerical tables, and among others, a multiplication table extending as far as 100 times 1000; tables of the squares of numbers, as far as 25,400; tables of cubes, and of the first ten powers of numbers, as far as 100.^[3] In 1814, Professor Barlow, of Woolwich, published, in an octavo volume, the squares, cubes, square roots, cube roots, and reciprocals of all numbers from 1 to 10,000; a table of the first ten powers of all numbers from 1 to 100, and of the fourth and fifth powers of all numbers from 100 to 1000.

^[2]Dodson's *Calculator*. 4to. London: 1747.

^[3]Hutton's *Tables of Products and Powers*. Folio. London; 1781.

Tables of Multiplication to a still greater extent have been published in France. In 1785, was published an octavo volume of tables of the squares, cubes, square roots, and cube roots of all numbers from 1 to 10,000; and similar tables were again published in 1801. In 1817, multiplication tables were published in Paris by Voisin; and similar tables, in two quarto volumes, in 1824, by the French Board of Longitude, extending as far as a thousand times a thousand. A table of squares was published in 1810, in Hanover; in 1812, at Leipzig; in 1825, at Berlin; and in 1827, at Ghent. A table of cubes was published in 1827, at Eisenach; in the same year a

similar table at Ghent; and one of the squares of all numbers as far as 10,000, was published in that year, in quarto, at Bonn. The Prussian Government has caused a multiplication table to be calculated and printed, extending as far as 1000 times 1000. Such are a few of the tables of this class which have been published in different countries.

This class of tables may be considered as purely arithmetical, since the results which they express involve no other relations than the arithmetical dependence of abstract numbers upon each other. When numbers, however, are taken in a concrete sense, and are applied to express peculiar modes of quantity,—such as angular, linear, superficial, and solid magnitudes,—a new set of numerical relations arise, and a large number of computations are required.

To express angular magnitude, and the various relations of linear magnitude with which it is connected, involves the consideration of a vast variety of Geometrical and Trigonometrical tables; such as tables of the natural sines, co-sines, tangents, secants, co-tangents, &c. &c.; tables of arcs and angles in terms of the radius; tables for the immediate solution of various cases of triangles, &c. Volumes without number of such tables have been from time to time computed and published. It is not sufficient, however, for the purposes of computation to tabulate these immediate trigonometrical functions. Their squares^[4] and higher powers, their square roots, and other roots, occur so frequently, that it has been found expedient to compute tables for them, as well as for the same functions of abstract numbers.

^[4]The squares of the sines of angles are extensively used in the calculations connected with the theory of the tides. Not aware that tables of these squares existed, Bouvard, who calculated the tides for Laplace, underwent the labour of calculating the square of each individual sine in every case in which it occurred.

The measurement of linear, superficial, and solid magnitudes, in the various forms and modifications in which they are required in the arts, demands another extensive catalogue of numerical tables. The surveyor, the architect, the builder, the carpenter, the miner, the ganger, the naval architect, the engineer, civil and military, all require the aid of peculiar numerical tables, and such have been published in all countries.

The increased expedition and accuracy which was introduced into the art of computation by the invention of Logarithms, greatly enlarged the number

of tables previously necessary. To apply the logarithmic method, it was not merely necessary to place in the hands of the computist extensive tables of the logarithms of the natural numbers, but likewise to supply him with tables in which he might find already calculated the logarithms of those arithmetical, trigonometrical, and geometrical functions of numbers, which he has most frequent occasion to use. It would be a circuitous process, when the logarithm of a sine or co-sine of an angle is required, to refer, first to the table of sines, or co-sines, and thence to the table of the logarithms of natural numbers. It was therefore found expedient to compute distinct tables of the logarithms of the sines, co-sines, tangents, &c., as well as of various other functions frequently required, such as sums, differences, &c.

Great as is the extent of the tables we have just enumerated, they bear a very insignificant proportion to those which remain to be mentioned. The above are, for the most part, general in their nature, not belonging particularly to any science or art. There is a much greater variety of tables, whose importance is no way inferior, which are, however, of a more special nature: Such are, for example, tables of interest, discount, and exchange, tables of annuities, and other tables necessary in life insurances; tables of rates of various kinds necessary in general commerce. But the science in which, above all others, the most extensive and accurate tables are indispensable, is Astronomy; with the improvement and perfection of which is inseparably connected that of the kindred art of Navigation. We scarcely dare hope to convey to the general reader any thing approaching to an adequate notion of the multiplicity and complexity of the tables necessary for the purposes of the astronomer and navigator. We feel, nevertheless, that the truly national importance which must attach to any perfect and easy means of producing those tables cannot be at all estimated, unless we state some of the previous calculations necessary in order to enable the mariner to determine, with the requisite certainty and precision, the place of his ship.

In a word, then, all the purely arithmetical, trigonometrical, and logarithmic tables already mentioned, are necessary, either immediately or remotely, for this purpose. But in addition to these, a great number of tables, exclusively astronomical, are likewise indispensable. The predictions of the astronomer, with respect to the positions and motions of the bodies of the firmament, are the means, and the only means, which enable the mariner to

prosecute his art. By these he is enabled to discover the distance of his ship from the Line, and the extent of his departure from the meridian of Greenwich, or from any other meridian to which the astronomical predictions refer. The more numerous, minute, and accurate these predictions can be made, the greater will be the facilities which can be furnished to the mariner. But the computation of those tables, in which the future position of celestial objects are registered, depend themselves upon an infinite variety of other tables which never reach the hands of the mariner. It cannot be said that there is any table whatever, necessary for the astronomer, which is unnecessary for the navigator.

The purposes of the marine of a country whose interests are so inseparably connected as ours are with the improvement of the art of navigation, would be very inadequately fulfilled, if our navigators were merely supplied with the means of determining by *Nautical Astronomy* the position of a ship at sea. It has been well observed by the Committee of the Astronomical Society, to whom the recent improvement of the Nautical Almanac was confided, that it is not by those means merely by which the seaman is enabled to determine the position of his vessel at sea, that the full intent and purpose of what is usually called *Nautical Astronomy* are answered. This object is merely a part of that comprehensive and important subject; and might be attained by a very cheap publication, and without the aid of expensive instruments. A not less important and much more difficult part of nautical science has for its object to determine the precise position of various interesting and important points on the surface of the earth,—such as remarkable headlands, ports, and islands; together with the general trending of the coast between well-known harbours. It is not necessary to point out here how important such knowledge is to the mariner. This knowledge, which may be called *Nautical Geography*, cannot be obtained by the methods of observation used on board ship, but requires much more delicate and accurate instruments, firmly placed upon the solid ground, besides all the astronomical aid which can be afforded by the best tables, arranged in the most convenient form for immediate use. This was Dr Maskelyne's view of the subject, and his opinion has been confirmed by the repeated wants and demands of those distinguished navigators who have been employed in several recent scientific expeditions.^[5]

^[5]Report of the Committee of the Astronomical Society prefixed to the Nautical Almanac for 1834.

Among the tables *directly* necessary for navigation, are those which predict the position of the centre of the sun from hour to hour. These tables include the sun's right ascension and declination, daily, at noon, with the hourly change in these quantities. They also include the equation of time, together with its hourly variation.

Tables of the moon's place for every hour, are likewise necessary, together with the change of declination for every ten minutes. The lunar method of determining the longitude depends upon tables containing the predicted distances of the moon from the sun, the principal planets, and from certain conspicuous fixed stars; which distances being observed by the mariner, he is enabled thence to discover the *time* at the meridian from which the longitude is measured; and, by comparing that time with the time known or discoverable in his actual situation, he infers his longitude. But not only does the prediction of the position of the moon, with respect to these celestial objects, require a vast number of numerical tables, but likewise the observations necessary to be made by the mariner, in order to determine the lunar distances, also require several tables. To predict the exact position of any fixed star, requires not less than ten numerical tables peculiar to that star; and if the mariner be furnished (as is actually the case) with tables of the predicted distances of the moon from one hundred such stars, such predictions must require not less than a thousand numerical tables. Regarding the range of the moon through the firmament, however, it will readily be conceived that a hundred stars form but a scanty supply; especially when it is considered that an accurate method of determining the longitude, consists in observing the extinction of a star by the dark edge of the moon. Within the limits of the lunar orbit there are not less than one thousand stars, which are so situated as to be in the moon's path, and therefore to exhibit, at some period or other, those desirable occultations. These stars are also of such magnitudes, that their occultations may be distinctly observed from the deck, even when subject to all the unsteadiness produced by an agitated sea. To predict the occultations of such stars, would require not less than ten thousand tables. The stars from which lunar distances might be taken are still more numerous; and we may safely pronounce, that, great as has been the improvement effected recently in our Nautical Almanac, it does not yet furnish more than a small fraction of that aid to navigation (in the large sense of that term), which, with greater

facility, expedition, and economy in the calculation and printing of tables, it might be made to supply.

Tables necessary to determine the places of the planets are not less necessary than those for the sun, moon, and stars. Some notion of the number and complexity of these tables may be formed, when we state that the positions of the two principal planets, (and these the most necessary for the navigator,) Jupiter and Saturn, require each not less than one hundred and sixteen tables. Yet it is not only necessary to predict the position of these bodies, but it is likewise expedient to tabulate the motions of the four satellites of Jupiter, to predict the exact times at which they enter his shadow, and at which their shadows cross his disc, as well as the times at which they are interposed between him and the Earth, and he between them and the Earth.

Among the extensive classes of tables here enumerated, there are several which are in their nature permanent and unalterable, and would never require to be recomputed, if they could once be computed with perfect accuracy on accurate data; but the data on which such computations are conducted, can only be regarded as approximations to truth, within limits the extent of which must necessarily vary with our knowledge of astronomical science. It has accordingly happened, that one set of tables after another has been superseded with each advance of astronomical science. Some striking examples of this may not be uninteresting. In 1765, the Board of Longitude paid to the celebrated Euler the sum of L.300, for furnishing general formulæ for the computation of lunar tables. Professor Mayer was employed to calculate the tables upon these formulæ, and the sum of L.3000 was voted for them by the British Parliament, to his widow, after his decease. These tables had been used for ten years, from 1766 to 1776, in computing the Nautical Almanac, when they were superseded by new and improved tables, composed by Mr Charles Mason, under the direction of Dr Maskelyne, from calculations made by order of the Board of Longitude, on the observations of Dr Bradley. A farther improvement was made by Mason in 1780; but a much more extensive improvement took place in the lunar calculations by the publication of the tables of the Moon, by M. Bürg, deduced from Laplace's theory, in 1806. Perfect, however, as Bürg's tables were considered, at the time of their publication, they were, within the short period of six years, superseded by a more accurate set of

tables published by Burckhardt in 1812; and these also have since been followed by the tables of Damoiseau. Professor Schumacher has calculated by the latter tables his ephemeris of the Planetary Lunar Distances, and astronomers will hence be enabled to put to the strict test of observation the merits of the tables of Burckhardt and Damoiseau.^[6]

^[6]A comparison of the results for 1834, will be found in the Nautical Almanac for 1835.

The solar tables have undergone, from time to time, similar changes. The solar tables of Mayer were used in the computation of the Nautical Almanac, from its commencement in 1767, to 1804 inclusive. Within the six years immediately succeeding 1804, not less than three successive sets of solar tables appeared, each improving on the other; the first by Baron de Zach, the second by Delambre, under the direction of the French Board of Longitude, and the third by Carlini. The last, however, differ only in arrangement from those of Delambre.

Similar observations will be applicable to the tables of the principal planets. Bouvard published, in 1803, tables of Jupiter and Saturn; but from the improved state of astronomy, he found it necessary to recompute these tables in 1821.

Although it is now about thirty years since the discovery of the four new planets, Ceres, Pallas, Juno, and Vesta, it was not till recently that tables of their motions were published. They have lately appeared in Encke's Ephemeris.

We have thus attempted to convey some notion (though necessarily a very inadequate one) of the immense extent of numerical tables which it has been found necessary to calculate and print for the purposes of the arts and sciences. We have before us a catalogue of the tables contained in the library of one private individual, consisting of not less than one hundred and forty volumes. Among these there are no duplicate copies: and we observe that many of the most celebrated voluminous tabular works are not contained among them. They are confined exclusively to arithmetical and trigonometrical tables; and, consequently, the myriad of astronomical and nautical tables are totally excluded from them. Nevertheless, they contain an extent of printed surface covered with figures amounting to above sixteen thousand square feet. We have taken at random forty of these tables,

and have found that the number of errors *acknowledged* in the respective errata, amounts to above *three thousand seven hundred*.

To be convinced of the necessity which has existed for accurate numerical tables, it will only be necessary to consider at what an immense expenditure of labour and of money even the imperfect ones which we possess have been produced.

To enable the reader to estimate the difficulties which attend the attainment even of a limited degree of accuracy, we shall now explain some of the expedients which have been from time to time resorted to for the attainment of numerical correctness in calculating and printing them.

Among the scientific enterprises which the ambition of the French nation aspired to during the Republic, was the construction of a magnificent system of numerical tables. Their most distinguished mathematicians were called upon to contribute to the attainment of this important object; and the superintendence of the undertaking was confided to the celebrated Prony, who co-operated with the government in the adoption of such means as might be expected to ensure the production of a system of logarithmic and trigonometric tables, constructed with such accuracy that they should form a monument of calculation the most vast and imposing that had ever been executed, or even conceived. To accomplish this gigantic task, the principle of the division of labour, found to be so powerful in manufactures, was resorted to with singular success. The persons employed in the work were divided into three sections: the first consisted of half a dozen of the most eminent analysts. Their duty was to investigate the most convenient mathematical formulæ, which should enable the computers to proceed with the greatest expedition and accuracy by the method of Differences, of which we shall speak more fully hereafter. These formulæ, when decided upon by this first section, were handed over to the second section, which consisted of eight or ten properly qualified mathematicians. It was the duty of this second section to convert into numbers certain general or algebraical expressions which occurred in the formulæ, so as to prepare them for the hands of the computers. Thus prepared, these formulæ were handed over to the third section, who formed a body of nearly one hundred computers. The duty of this numerous section was to compute the numbers finally intended for the tables. Every possible precaution was of course taken to ensure the numerical accuracy of the results. Each number was calculated by two or

more distinct and independent computers, and its truth and accuracy determined by the coincidence of the results thus obtained.

The body of tables thus calculated occupied in manuscript *seventeen* folio volumes.^[7]

^[7]These tables were never published. The printing of them was commenced by Didot, and a small portion was actually stereotyped, but never published. Soon after the commencement of the undertaking, the sudden fall of the assignats rendered it impossible for Didot to fulfil his contract with the government. The work was accordingly abandoned, and has never since been resumed. We have before us a copy of 100 pages folio of the portion which was printed at the time the work was stopped, given to a friend on a late occasion by Didot himself. It was remarked in this, as in other similar cases, that the computers who committed fewest errors were those who understood nothing beyond the process of addition.

As an example of the precautions which have been considered necessary to guard against errors in the calculation of numerical tables, we shall further state those which were adopted by Mr Babbage, previously to the publication of his tables of logarithms. In order to render the terminal figure of tables in which one or more decimal places are omitted as accurate as it can be, it has been the practice to compute one or more of the succeeding figures; and if the first omitted figure be greater than 4, then the terminal figure is always increased by 1, since the value of the tabulated number is by such means brought nearer to the truth. ^[8] The tables of Callet, which were among the most accurate published logarithms, and which extended to seven places of decimals, were first carefully compared with the tables of Vega, which extended to ten places, in order to discover whether Callet had made the above correction of the final figure in every case where it was necessary. This previous precaution being taken, and the corrections which appeared to be necessary being made in a copy of Callet's tables, the proofs of Mr Babbage's tables were submitted to the following test: They were first compared, number by number, with the corrected copy of Callet's logarithms; secondly, with Hutton's logarithms; and thirdly, with Vega's logarithms. The corrections thus suggested being marked in the proofs, corrected revises were received back. These revises were then again compared, number by number, first with Vega's logarithms; secondly, with the logarithms of Callet; and thirdly, as far as the first 20,000 numbers, with the corresponding ones in Briggs's logarithms. They were now returned to the printer, and were stereotyped; proofs were taken from the stereotyped

plates, which were put through the following ordeal: They were first compared once more with the logarithms of Vega as far as 47,500; they were then compared with the whole of the logarithms of Gardner; and next with the whole of Taylor's logarithms; and as a last test, they were transferred to the hands of a different set of readers, and were once more compared with Taylor. That these precautions were by no means superfluous may be collected from the following circumstances mentioned by Mr Babbage: In the sheets read immediately previous to stereotyping, thirty-two errors were detected; after stereotyping, eight more were found, and corrected in the plates.

[8] Thus suppose the number expressed at full length were 3.1415927. If the table extend to no more than four places of decimals, we should tabulate the number 3.1416 and not 3.1415. The former would be evidently nearer to the true number 3.1415927.

By such elaborate and expensive precautions many of the errors of computation and printing may certainly be removed; but it is too much to expect that in general such measures can be adopted; and we accordingly find by far the greater number of tables disfigured by errors, the extent of which is rather to be conjectured than determined. When the nature of a numerical table is considered,—page after page densely covered with figures, and with nothing else,—the chances against the detection of any single error will be easily comprehended; and it may therefore be fairly presumed, that for one error which may happen to be detected, there must be a great number which escape detection. Notwithstanding this difficulty, it is truly surprising how great a number of numerical errors have been detected by individuals no otherwise concerned in the tables than in their use. Mr Baily states that he has himself detected in the solar and lunar tables, from which our Nautical Almanac was for a long period computed, more than five hundred errors. In the multiplication table already mentioned, computed by Dr Hutton for the Board of Longitude, a single page was examined and recomputed: it was found to contain about forty errors.

In order to make the calculations upon the numbers found in the Ephemeral Tables published in the Nautical Almanac, it is necessary that the mariner should be supplied with certain permanent tables. A volume of these, to the number of about thirty, was accordingly computed, and published at national expense, by order of the Board of Longitude, entitled

'Tables requisite to be used with the Nautical Ephemeris for finding the latitude and longitude at sea.' In the first edition of these requisite tables, there were detected, by one individual, above a thousand errors.

The tables published by the Board of Longitude for the correction of the observed distances of the moon from certain fixed stars, are followed by a table of acknowledged errata, extending to seven folio pages, and containing more than eleven hundred errors. Even this table of errata itself is not correct: a considerable number of errors have been detected in it, so that errata upon errata have become necessary.

One of the tests most frequently resorted to for the detection of errors in numerical tables, has been the comparison of tables of the same kind, published by different authors. It has been generally considered that those numbers in which they are found to agree must be correct; inasmuch as the chances are supposed to be very considerable against two or more independent computers falling into precisely the same errors. How far this coincidence may be safely assumed as a test of accuracy we shall presently see.

A few years ago, it was found desirable to compute some very accurate logarithmic tables for the use of the great national survey of Ireland, which was then, and still is in progress; and on that occasion a careful comparison of various logarithmic tables was made. Six remarkable errors were detected, which were found to be common to several apparently independent sets of tables. This singular coincidence led to an unusually extensive examination of the logarithmic tables published both in England and in other countries; by which it appeared that thirteen sets of tables, published in London between the years 1633 and 1822, all agreed in these six errors. Upon extending the enquiry to foreign tables, it appeared that two sets of tables published at Paris, one at Gouda, one at Avignon, one at Berlin, and one at Florence, were infected by exactly the same six errors. The only tables which were found free from them were those of Vega, and the more recent impressions of Callet. It happened that the Royal Society possessed a set of tables of logarithms printed in the Chinese character, and on Chinese paper, consisting of two volumes: these volumes contained no indication or acknowledgment of being copied from any other work. They were examined; and the result was the detection in them of the same six errors.^[9]

It is quite apparent that this remarkable coincidence of error must have arisen from the various tables being copied successively one from another. The earliest work in which they appeared was Vlacq's Logarithms, (folio, Gouda, 1628); and from it, doubtless, those which immediately succeeded it in point of time were copied; from which the same errors were subsequently transcribed into all the other, including the Chinese logarithms.

The most certain and effectual check upon errors which arise in the process of computation, is to cause the same computations to be made by separate and independent computers; and this check is rendered still more decisive if they make their computations by different methods. It is, nevertheless, a remarkable fact, that several computers, working separately and independently, do frequently commit precisely the same error; so that falsehood in this case assumes that character of consistency, which is regarded as the exclusive attribute of truth. Instances of this are familiar to most persons who have had the management of the computation of tables. We have reason to know, that M. Prony experienced it on many occasions in the management of the great French tables, when he found three, and even a greater number of computers, working separately and independently, to return him the same numerical result, and *that result wrong*. Mr Stratford, the conductor of the Nautical Almanac, to whose talents and zeal that work owes the execution of its recent improvements, has more than once observed a similar occurrence. But one of the most signal examples of this kind, of which we are aware, is related by Mr Baily. The catalogue of stars published by the Astronomical Society was computed by two separate and independent persons, and was afterwards compared and examined with great care and attention by Mr Stratford. On examining this catalogue, and recalculating a portion of it, Mr Baily discovered an error in the case of the star, χ Cephei. Its right ascension was calculated *wrongly*, and yet *consistently*, by two computers working separately. Their numerical results agreed precisely in every figure; and Mr Stratford, on examining the catalogue, failed to detect the error. Mr Baily having reason, from some discordancy which he observed, to suspect an error, recomputed the place of the star with a view to discover it; and he himself, in the first instance, obtained precisely *the same erroneous numerical result*. It was only on

going over the operation a second time that he *accidentally* discovered that he had inadvertently committed the same error.^[10]

^[10]Memoirs Ast. Soc. vol. iv., p. 290.

It appears, therefore, that the coincidence of different tables, even when it is certain that they could not have been copied one from another, but must have been computed independently, is not a decisive test of their correctness, neither is it possible to ensure accuracy by the device of separate and independent computation.

Besides the errors incidental to the process of computation, there are further liabilities in the process of transcribing the final results of each calculation into the fair copy of the table designed for the printer. The next source of error lies with the compositor, in transferring this copy into type. But the liabilities to error do not stop even here; for it frequently happens, that after the press has been fully corrected, errors will be produced in the process of printing. A remarkable instance of this occurs in one of the six errors detected in so many different tables already mentioned. In one of these cases, the last five figures of two successive numbers of a logarithmic table were the following:—

35875
10436.

Now, both of these are erroneous; the figure 8 in the first line should be 4, and the figure 4 in the second should be 8. It is evident that the types, as first composed, were correct; but in the course of printing, the two types 4 and 8 being loose, adhered to the inking-balls, and were drawn out: the pressmen in replacing them transposed them, putting the 8 *above* and the 4 *below*, instead of *vice versa*. It would be a curious enquiry, were it possible to obtain all the copies of the original edition of Vlacq's Logarithms, published at Gouda in 1628, from which this error appears to have been copied in all the subsequent tables, to ascertain whether it extends through the entire edition. It would probably, nay almost certainly, be discovered that some of the copies of that edition are correct in this number, while others are incorrect; the former having been worked off before the transposition of the types.

It is a circumstance worthy of notice, that this error in Vlacq's tables has produced a corresponding error in a variety of other tables deduced from them, *in which nevertheless the erroneous figures in Vlacq are omitted*. In no less than sixteen sets of tables published at various times since the publication of Vlacq, in which the logarithms extend only to seven places of figures, the error just mentioned in the *eighth place* in Vlacq causes a corresponding error in the *seventh place*. When the last three figures are omitted in the first of the above numbers, the seventh figure should be 5, inasmuch as the first of the omitted figures is under 5: the erroneous insertion, however, of the figure 8 in Vlacq has caused the figure 6 to be substituted for 5 in the various tables just alluded to. For the same reason, the erroneous occurrence of 4 in the second number has caused the adoption of a 0 instead of a 1 in the seventh place in the other tables. The only tables in which this error does not occur are those of Vega, the more recent editions of Callet, and the still later Logarithms of Mr Babbage.

The *Opus Palatinum*, a work published in 1596, containing an extensive collection of trigonometrical tables, affords a remarkable instance of a tabular error; which, as it is not generally known, it may not be uninteresting to mention here. After that work had been for several years in circulation in every part of Europe, it was discovered that the commencement of the table of co-tangents and co-secants was vitiated by an error of considerable magnitude. In the first co-tangent the last nine places of figures were incorrect; but from the manner in which the numbers of the table were computed, the error was gradually, though slowly, diminished, until at length it became extinguished in the eighty-sixth page. After the detection of this extensive error, Pitiscus undertook the recomputation of the eighty-six erroneous pages. His corrected calculation was printed, and the erroneous part of the remaining copies of the *Opus Palatinum* was cancelled. But as the corrected table of Pitiscus was not published until 1607,—thirteen years after the original work,—the erroneous part of the volume was cancelled in comparatively few copies, and consequently correct copies of the work are now exceedingly rare. Thus, in the collection of tables published by M. Schulze,^[11] the whole of the erroneous part of the *Opus Palatinum* has been adopted; he having used the copy of that work which exists in the library of the Academy of Berlin, and which is one of those copies in which the incorrect part was not cancelled. The corrected copies of this work may be very easily

distinguished at present from the erroneous ones: it happened that the former were printed with a very bad and worn-out type, and upon paper of a quality inferior to that of the original work. On comparing the first eighty-six pages of the volume with the succeeding ones, they are, therefore, immediately distinguishable in the corrected copies. Besides this test, there is another, which it may not be uninteresting to point out:—At the bottom of page 7 in the corrected copies, there is an error in the position of the words *basis* and *hypothenua*, their places being interchanged. In the original uncorrected work this error does not exist.

[11] *Recueil des Tables Logarithmiques et Trigonometriques*. Par J. C. Schulze. 2 vols. Berlin: 1778.

At the time when the calculation and publication of Taylor's Logarithms were undertaken, it so happened that a similar work was in progress in France; and it was not until the calculation of the French work was completed, that its author was informed of the publication of the English work. This circumstance caused the French calculator to relinquish the publication of his tables. The manuscript subsequently passed into the library of Delambre, and, after his death, was purchased at the sale of his books, by Mr Babbage, in whose possession it now is. Some years ago it was thought advisable to compare these manuscript tables with Taylor's Logarithms, with a view to ascertain the errors in each, but especially in Taylor. The two works were peculiarly well suited for the attainment of this end; as the circumstances under which they were produced, rendered it quite certain that they were computed independently of each other. The comparison was conducted under the direction of the late Dr Young, and the result was the detection of the following nineteen errors in Taylor's Logarithms. To enable those who used Taylor's Logarithms to make the necessary corrections in them, the corrections of the detected errors appeared as follows in the Nautical Almanac for 1832.

ERRATA, *detected in Taylor's Logarithms*. London: 4to, 1792.

			o	'	"	
1	<i>E</i>	Co-tangent of	1.35.35	for 43671	read 42671	
2	<i>M</i>	Co-tangent of	4. 4.49	— 66976	— 66979	
3		Sine of	4.23.38	— 43107	— 43007	
4		Sine of	4.23.39	— 43381	— 43281	
5	<i>S</i>	Sine of	6.45.52	— 10001	— 11001	
6	<i>Kk</i>	Co-sine of	14.18. 3	— 3398	— 3298	
7	<i>Ss</i>	Tangent of	18. 1.56	— 5064	— 6064	

8	<i>Aaa</i>	Co-tangent of	21.11.14	— 6062	—— 5962
9	<i>Ggg</i>	Tangent of	23.48.19	— 6087	—— 5987
10		Co-tangent of	23.48.19	— 3913	—— 4013
11	<i>Iii</i>	Sine of	25. 5. 4	— 3173	—— 3183
12		Sine of	25. 5. 5	— 3218	—— 3228
13		Sine of	25. 5. 6	— 3263	—— 3273
14		Sine of	25. 5. 7	— 3308	—— 3318
15		Sine of	25. 5. 8	— 3353	—— 3363
16		Sine of	25. 5. 9	— 3398	—— 3408
17	<i>Qqq</i>	Tangent of	28.19.39	— 6302	—— 6402
18	<i>4H</i>	Tangent of	35.55.51	— 1681	—— 1581
19	<i>4K</i>	Co-sine of	37.29. 2	— 5503	—— 5603

An error being detected in this list of ERRATA, we find, in the Nautical Almanac for the year 1833, the following ERRATUM of the ERRATA of Taylor's Logarithms:—

'In the list of ERRATA detected in Taylor's Logarithms, for *cos.* 4° 18' 3", read *cos.* 14° 18' 2".'

Here, however, confusion is worse confounded; for a new error, not before existing, and of much greater magnitude, is introduced! It will be necessary, in the Nautical Almanac for 1836, (that for 1835 is already published,) to introduce the following:

ERRATUM of the ERRATUM of the ERRATA of TAYLOR's *Logarithms*. For *cos.* 4° 18' 3", *read cos.* 14° 18' 3".

If proof were wanted to establish incontrovertibly the utter impracticability of precluding numerical errors in works of this nature, we should find it in this succession of error upon error, produced, in spite of the universally acknowledged accuracy and assiduity of the persons at present employed in the construction and management of the Nautical Almanac. It is only by the *mechanical fabrication of tables* that such errors can be rendered impossible.

On examining this list with attention, we have been particularly struck with the circumstances in which these errors appear to have originated. It is a remarkable fact, that of the above nineteen errors, eighteen have arisen from mistakes in *carrying*. Errors 5, 7, 10, 11, 12, 13, 14, 15, 16, 17, 19, have arisen from a carriage being neglected; and errors 1, 3, 4, 6, 8, 9, and 18, from a carriage being made where none should take place. In four cases, namely, errors 8, 9, 10, and 16, this has caused *two* figures to be wrong. The only error of the nineteen which appears to have been a press error is the second; which has evidently arisen from the type 9 being accidentally inverted, and thus becoming a 6. This may have originated with the compositor, but more probably it took place in the press-work; the type 9 being accidentally drawn out of the form by the inking-ball, as mentioned in a former case, and on being restored to its place, inverted by the pressman.

There are two cases among the above errata, in which an error, committed in the calculation of one number, has evidently been the cause of other errors. In the third erratum, a wrong carriage was made, in computing the sine of $4^{\circ} 23' 38''$. The next number of the table was vitiated by this error; for we find the next erratum to be in the sine of $4^{\circ} 23' 39''$, in which the figure similarly placed is 1 in excess. A still more extensive effect of this kind appears in errata 11, 12, 13, 14, 15, 16. A carriage was neglected in computing the sine of $25^{\circ} 5' 4''$, and this produced a corresponding error in the five following numbers of the table, which are those corrected in the five following errata.

This frequency of errors arising in the process of carrying, would afford a curious subject of metaphysical speculation respecting the operation of the faculty of memory. In the arithmetical process, the memory is employed in a twofold way;—in ascertaining each successive figure of the calculated result by the recollection of a table committed to memory at an early period of life; and by another act of memory, in which the number carried from column to column is retained. It is a curious fact, that this latter circumstance, occurring only the moment before, and being in its nature little complex, is so much more liable to be forgotten or mistaken than the results of rather complicated tables. It appears, that among the above errata, the errors 5, 7, 10, 11, 17, 19, have been produced by the computer forgetting a carriage; while the errors 1, 3, 6, 8, 9, 18, have been produced by his making a carriage improperly. Thus, so far as the above list of errata

affords grounds for judging, it would seem, (contrary to what might be expected,) that the error by which improper carriages are made is as frequent as that by which necessary carriages are overlooked.

We trust that we have succeeded in proving, first, the great national and universal utility of numerical tables, by showing the vast number of them, which have been calculated and published; secondly, that more effectual means are necessary to obtain such tables suitable to the present state of the arts, sciences and commerce, by showing that the existing supply of tables, vast as it certainly is, is still scanty, and utterly inadequate to the demands of the community;—that it is rendered inefficient, not only in quantity, but in quality, by its want of numerical correctness; and that such numerical correctness is altogether unattainable until some more perfect method be discovered, not only of calculating the numerical results, but of tabulating these,—of reducing such tallies to type, and of printing that type so as to intercept the possibility of error during the press-work. Such are the ends which are proposed to be attained by the calculating machinery invented by Mr Babbage.

The benefits to be derived from this invention cannot be more strongly expressed than they have been by Mr Colebrooke, President of the Astronomical Society, on the occasion of presenting the gold medal voted by that body to Mr Babbage:—'In no department of science, or of the arts, does this discovery promise to be so eminently useful as in that of astronomy, and its kindred sciences, with the various arts dependent on them. In none are computations more operose than those which astronomy in particular requires;—in none are preparatory facilities more needful;—in none is error more detrimental. The practical astronomer is interrupted in his pursuit, and diverted from his task of observation by the irksome labours of computation, or his diligence in observing becomes ineffectual for want of yet greater industry of calculation. Let the aid which tables previously computed afford, be furnished to the utmost extent which mechanism has made attainable through Mr Babbage's invention, and the most irksome portion of the astronomer's task is alleviated, and a fresh impulse is given to astronomical research.'

The first step in the progress of this singular invention was the discovery of some common principle which pervaded numerical tables of every description; so that by the adoption of such a principle as the basis of the machinery, a corresponding degree of generality would be conferred upon its calculations. Among the properties of numerical functions, several of a general nature exist; and it was a matter of no ordinary difficulty, and requiring no common skill, to select one which might, in all respects, be preferable to the others. Whether or not that which was selected by Mr Babbage affords the greatest practical advantages, would be extremely difficult to decide—perhaps impossible, unless some other projector could be found possessed of sufficient genius, and sustained by sufficient energy of mind and character, to attempt the invention of calculating machinery on other principles. The principle selected by Mr Babbage as the basis of that part of the machinery which calculates, is the Method of Differences; and he has in fact literally thrown this mathematical principle into wheel-work. In order to form a notion of the nature of the machinery, it will be necessary, first to convey to the reader some idea of the mathematical principle just alluded to.

A numerical table, of whatever kind, is a series of numbers which possess some common character, and which proceed increasing or decreasing according to some general law. Supposing such a series continually to increase, let us imagine each number in it to be subtracted from that which follows it, and the remainders thus successively obtained to be ranged beside the first, so as to form another table: these numbers are called the *first differences*. If we suppose these likewise to increase continually, we may obtain a third table from them by a like process, subtracting each number from the succeeding one: this series is called the *second differences*. By adopting a like method of proceeding, another series may be obtained, called the *third differences*; and so on. By continuing this process, we shall at length obtain a series of differences, of some order, more or less high, according to the nature of the original table, in which we shall find the same number constantly repeated, to whatever extent the original table may have been continued; so that if the next series of differences had been obtained in the same manner as the preceding ones, every term of it would be 0. In some cases this would continue to whatever extent the original table might be carried; but in all cases a series of

differences would be obtained, which would continue constant for a very long succession of terms.

As the successive serieses of differences are derived from the original table, and from each other, by *subtraction*, the same succession of series may be reproduced in the other direction by *addition*. But let us suppose that the first number of the original table, and of each of the series of differences, including the last, be given: all the numbers of each of the series may thence be obtained by the mere process of addition. The second term of the original table will be obtained by adding to the first the first term of the first difference series; in like manner, the second term of the first difference series will be obtained by adding to the first term, the first term of the third difference series, and so on. The second terms of all the serieses being thus obtained, the third terms may be obtained by a like process of addition; and so the series may be continued. These observations will perhaps be rendered more clearly intelligible when illustrated by a numerical example. The following is the commencement of a series of the fourth powers of the natural numbers:—

No.	Table.
1	1
2	16
3	81
4	256
5	625
6	1296
7	2401
8	4096
9	6561
10	10,000
11	14,641
12	20,736
13	28,561

By subtracting each number from the succeeding one in this series, we obtain the following series of first differences:

15
65
175
369
671
1105
1695
2465
3439
4641
6095
7825

In like manner, subtracting each term of this series from the succeeding one, we obtain the following series of second differences:—

50
110
194
302
434
590
770
974
1202
1454
1730

Proceeding with this series in the same way, we obtain the following series of third differences:—

60
84
108
132
156
180
204
228

252
276

Proceeding in the same way with these, we obtain the following for the series of fourth differences:—

24
24
24
24
24
24
24
24
24

It appears, therefore, that in this case the series of fourth differences consists of a constant repetition of the number 24. Now, a slight consideration of the succession of arithmetical operations by which we have obtained this result, will show, that by reversing the process, we could obtain the table of fourth powers by the mere process of addition. Beginning with the first numbers in each successive series of differences, and designating the table and the successive differences by the letters T, D¹ D² D³ D⁴, we have then the following to begin with:—

T	D ¹	D ²	D ³	D ⁴
1	15	50	60	24

Adding each number to the number on its left, and repeating 24, we get the following as the second terms of the several series:—

T	D ¹	D ²	D ³	D ⁴
16	65	110	84	24

And, in the same manner, the third and succeeding terms as follows:—

No.	T	D ¹	D ²	D ³	D ⁴
1	1	15	50	60	24
2	16	65	110	84	24

No.	T	D ¹	D ²	D ³	D ⁴
3	81	175	194	108	24
4	256	369	302	132	24
5	625	671	434	156	24
6	1296	1105	590	180	24
7	2401	1695	770	204	24
8	4096	2465	974	228	24
9	6561	3439	1202	252	24
10	10000	4641	1454	276	
11	14641	6095	1730		
12	20736	7825			
13	28561				

There are numerous tables in which, as already stated, to whatever order of differences we may proceed, we should not obtain a series of rigorously constant differences; but we should always obtain a certain number of differences which to a given number of decimal places would remain constant for a long succession of terms. It is plain that such a table might be calculated by addition in the same manner as those which have a difference rigorously and continuously constant; and if at every point where the last difference requires an increase, that increase be given to it, the same principle of addition may again be applied for a like succession of terms, and so on.

By this principle it appears, that all tables in which each series of differences continually increases, may be produced by the operation of addition alone; provided the first terms of the table, and of each series of differences, be given in the first instance. But it sometimes happens, that while the table continually increases, one or more serieses of differences may continually diminish. In this case, the series of differences are found by subtracting each term of the series, not from that which follows, but from that which precedes it; and consequently, in the re-production of the several serieses, when their first terms are given, it will be necessary in some cases to obtain them by *addition*, and in others by *subtraction*. It is possible, however, still to perform all the operations by addition alone: this is

effected in performing the operation of subtraction, by substituting for the subtrahend its *arithmetical complement*, and adding that, omitting the unit of the highest order in the result. This process, and its principle, will be readily comprehended by an example. Let it be required to subtract 357 from 768.

The common process would be as follows:—

From	768
Subtract	357
Remainder	411

The *arithmetical complement* of 357, or the number by which it falls short of 1000, is 643. Now, if this number be added to 768, and the first figure on the left be struck out of the sum, the process will be as follows:—

To	768
Add	643
Sum	1411
Remainder sought	411

The principle on which this process is founded is easily explained. In the latter process we have first added 643, and then subtracted 1000. On the whole, therefore, we have subtracted 357, since the number actually subtracted exceeds the number previously added by that amount.

Since, therefore, subtraction may be effected in this manner by addition, it follows that the calculation of all serieses, so far as an order of differences can be found in them which continues constant, may be conducted by the process of addition alone.

It also appears from what has been stated, that each addition consists only of two operations. However numerous the figures may be of which the several pairs of numbers to be thus added may consist, it is obvious that the operation of adding them can only consist of repetitions of the process of adding one digit to another; and of carrying one from the column of inferior

units to the column of units next superior when necessary. If we would therefore reduce such a process to machinery, it would only be necessary to discover such a combination of moving parts as are capable of performing these two processes of *adding* and *carrying* on two single figures; for, this being once accomplished, the process of adding two numbers, consisting of any number of digits, will be effected by repeating the same mechanism as often as there are pairs of digits to be added. Such was the simple form to which Mr Babbage reduced the problem of discovering the calculating machinery; and we shall now proceed to convey some notion of the manner in which he solved it.

For the sake of illustration, we shall suppose that the table to be calculated shall consist of numbers not exceeding six places of figures; and we shall also suppose that the difference of the fifth order is the constant difference. Imagine, then, six rows of wheels, each wheel carrying upon it a dial-plate like that of a common clock, but consisting of *ten* instead of *twelve* divisions; the several divisions being marked 1, 2, 3, 4, 5, 6, 7, 8, 9, 0. Let these dials be supposed to revolve whenever the wheels to which they are attached are put in motion, and to turn in such a direction that the series of increasing numbers shall pass under the index which appears over each dial:—thus, after 0 passes the index, 1 follows, then 2, 3, and so on, as the dial revolves. In Fig. 1 are represented six horizontal rows of such dials.

fig01

Fig. 1.

The method of differences, as already explained, requires, that in proceeding with the calculation, this apparatus should perform continually the addition of the number expressed upon each row of dials, to the number expressed upon the row immediately above it. Now, we shall first explain how this process of addition may be conceived to be performed by the motion of the dials; and in doing so, we shall consider separately the processes of addition and carriage, considering the addition first, and then the carriage.

Let us first suppose the line D^1 to be added to the line T. To accomplish this, let us imagine that while the dials on the line D^1 are quiescent, the dials on the line T are put in motion, in such a manner, that as many

divisions on each dial shall pass under its index, as there are units in the number at the index immediately below it. It is evident that this condition supposes, that if 0 be at any index on the line D^1 , the dial immediately above it in the line T shall not move. Now the motion here supposed, would bring under the indices on the line T such a number as would be produced by adding the number D^1 to T, neglecting all the carriages; for a carriage should have taken place in every case in which the figure 9 of any dial in the line T had passed under the index during the adding motion. To accomplish this carriage, it would be necessary that the dial immediately on the left of any dial in which 9 passes under the index, should be advanced one division, independently of those divisions which it may have been advanced by the addition of the number immediately below it. This effect may be conceived to take place in either of two ways. It may be either produced at the moment when the division between 9 and 0 of any dial passes under the index; in which case the process of carrying would go on simultaneously with the process of adding; or the process of carrying may be postponed in every instance until the process of addition, without carrying, has been completed; and then by another distinct and independent motion of the machinery, a carriage may be made by advancing one division all those dials on the right of which a dial had, during the previous addition, passed from 9 to 0 under the index. The latter is the method adopted in the calculating machinery, in order to enable its inventor to construct the carrying machinery independent of the adding mechanism.

Having explained the motion of the dials by which the addition, excluding the carriages of the number on the row D^1 , may be made to the number on the row T, the same explanation may be applied to the number on the row D^2 to the number on the row D^1 ; also, of the number 3 to the number on the row 2 , and so on. Now it is possible to suppose the additions of all the rows, except the first, to be made to all the rows except the last, simultaneously; and after these additions have been made, to conceive all the requisite carriages to be also made by advancing the proper dials one division forward. This would suppose all the dials in the scheme to receive their adding motion together; and, this being accomplished, the requisite dials to receive their carrying motions together. The production of so great a number of simultaneous motions throughout any machinery, would be attended with great mechanical difficulties, if indeed it be practicable. In the

calculating machinery it is not attempted. The additions are performed in two successive periods of time, and the carriages in two other periods of time, in the following manner. We shall suppose one complete revolution of the axis which moves the machinery, to make one complete set of additions and carriages; it will then make them in the following order:—

The first quarter of a turn of the axis will add the second, fourth, and sixth rows to the first, third, and fifth, omitting the carriages; this it will do by causing the dials on the first, third, and fifth rows, to turn through as many divisions as are expressed by the numbers at the indices below them, as already explained.

The second quarter of a turn will cause the carriages consequent on the previous addition, to be made by moving forward the proper dials one division.

(During these two quarters of a turn, the dials of the first, third, and fifth row alone have been moved; those of the second, fourth, and sixth, have been quiescent.)

The third quarter of a turn will produce the addition of the third and fifth rows to the second and fourth, omitting the carriages; which it will do by causing the dials of the second and fourth rows to turn through as many divisions as are expressed by the numbers at the indices immediately below them.

The fourth and last quarter of a turn will cause the carriages consequent on the previous addition, to be made by moving the proper dials forward one division.

This evidently completes one calculation, since all the rows except the first have been respectively added to all the rows except the last.

To illustrate this: let us suppose the table to be computed to be that of the fifth powers of the natural numbers, and the computation to have already proceeded so far as the fifth power of 6, which is 7776. This number appears, accordingly, in the highest row, being the place appropriated to the number of the table to be calculated. The several differences as far as the fifth, which is in this case constant, are exhibited on the successive rows of dials in such a manner, as to be adapted to the process of addition by alternate rows, in the manner already explained. The process of addition

will commence by the motion of the dials in the first, third, and fifth rows, in the following manner: The dial A, [fig. 1](#), must turn through one division, which will bring the number 7 to the index; the dial B must turn through three divisions, which will 0 bring to the index; this will render a carriage necessary, but that carriage will not take place during the present motion of the dial. The dial C will remain unmoved, since 0 is at the index below it; the dial D must turn through nine divisions; and as, in doing so, the division between 9 and 0 must pass under the index, a carriage must subsequently take place upon the dial to the left; the remaining dials of the row T, [fig. 1](#), will remain unmoved. In the row D^2 the dial A^2 will remain unmoved, since 0 is at the index below it; the dial B^2 will be moved through five divisions, and will render a subsequent carriage on the dial to the left necessary; the dial C^2 will be moved through five divisions; the dial D^2 will be moved through three divisions, and the remaining dials of this row will remain unmoved. The dials of the row D^4 will be moved according to the same rules; and the whole scheme will undergo a change exhibited in [fig. 2](#); a mark (*) being introduced on those dials to which a carriage is rendered necessary by the addition which has just taken place.

fig02

Fig. 2.

The second quarter of a turn of the moving axis, will move forward through one division all the dials which in [fig. 2](#) are marked (*), and the scheme will be converted into the scheme expressed in [fig. 3](#).

fig03

Fig. 3.

In the third quarter of a turn, the dial A^1 , [fig. 3](#), will remain unmoved, since 0 is at the index below it; the dial B^1 will be moved forward through three divisions; C^1 through nine divisions, and so on; and in like manner the dials of the row D^3 will be moved forward through the number of divisions expressed at the indices in the row D^4 . This change will convert the

arrangement into that expressed in [fig. 4](#), the dials to which a carriage is due, being distinguished as before by (*).

fig04

Fig. 4.

The fourth quarter of a turn of the axis will move forward one division all the dials marked (*); and the arrangement will finally assume the form exhibited in [fig. 5](#), in which the calculation is completed. The first row T in this expresses the fifth power of 7; and the second expresses the number which must be added to the first row, in order to produce the fifth power of 8; the numbers in each row being prepared for the change which they must undergo, in order to enable them to continue the computation according to the method of alternate addition here adopted.

fig05

Fig. 5.

Having thus explained what it is that the mechanism is required to do, we shall now attempt to convey at least a general notion of some of the mechanical contrivances by which the desired ends are attained. To simplify the explanation, let us first take one particular instance—the dials B and B¹, [fig. 1](#), for example. Behind the dial B¹ is a bolt, which, at the commencement of the process, is shot between the teeth of a wheel which drives the dial B: during the first quarter of a turn this bolt is made to revolve, and if it continued to be engaged in the teeth of the said wheel, it would cause the dial B to make a complete revolution; but it is necessary that the dial B should only move through three divisions, and, therefore, when three divisions of this dial have passed under its index, the aforesaid bolt must be withdrawn: this is accomplished by a small wedge, which is placed in a fixed position on the wheel behind the dial B¹, and that position is such that this wedge will press upon the bolt in such a manner, that at the moment when three divisions of the dial B have passed under the index, it shall withdraw the bolt from the teeth of the wheel which it drives. The bolt will continue to revolve during the remainder of the first quarter of a turn of the axis, but it will no longer drive the dial B, which will remain quiescent.

Had the figure at the index of the dial B^1 been any other, the wedge which withdraws the bolt would have assumed a different position, and would have withdrawn the bolt at a different time, but at a time always corresponding with the number under the index of the dial B^1 : thus, if 5 had been under the index of the dial B^1 , then the bolt would have been withdrawn from between the teeth of the wheel which it drives, when five divisions of the dial B had passed under the index, and so on. Behind each dial in the row D^1 there is a similar bolt and a similar withdrawing wedge, and the action upon the dial above is transmitted and suspended in precisely the same manner. Like observations will be applicable to all the dials in the scheme here referred to, in reference to their adding actions upon those above them.

There is, however, a particular case which here merits notice: it is the case in which 0 is under the index of the dial from which the addition is to be transmitted upwards. As in that case nothing is to be added, a mechanical provision should be made to prevent the bolt from engaging in the teeth of the wheel which acts upon the dial above: the wedge which causes the bolt to be withdrawn, is thrown into such a position as to render it impossible that the bolt should be shot, or that it should enter between the teeth of the wheel, which in other cases it drives. But inasmuch as the usual means of shooting the bolt would still act, a strain would necessarily take place in the parts of the mechanism, owing to the bolt not yielding to the usual impulse. A small shoulder is therefore provided, which puts aside, in this case, the piece by which the bolt is usually struck, and allows the striking implement to pass without encountering the head of the bolt or any other obstruction. This mechanism is brought into play in the scheme, [fig. 1](#), in the cases of all those dials in which 0 is under the index.

Such is a general description of the nature of the mechanism by which the adding process, apart from the carriages, is effected. During the first quarter of a turn, the bolts which drive the dials in the first, third, and fifth rows, are caused to revolve, and to act upon these dials, so long as they are permitted by the position of the several wedges on the second, fourth, and sixth rows of dials, by which these bolts are respectively withdrawn; and, during the third quarter of a turn, the bolts which drive the dials of the second and fourth rows are made to revolve and act upon these dials so long as the wedges on the dials of the third and fifth rows, which withdraw them,

permit. It will hence be perceived, that, during the first and third quarters of a turn, the process of addition is continually passing upwards through the machinery; alternately from the even to the odd rows, and from the odd to the even rows, counting downwards.

We shall now attempt to convey some notion of the mechanism by which the process of carrying is effected during the second and fourth quarters of a turn of the axis. As before, we shall first explain it in reference to a particular instance. During the first quarter of a turn the wheel B^2 , [fig. 1](#), is caused by the adding bolt to move through five divisions; and the fifth of these divisions, which passes under the index, is that between 9 and 0. On the axis of the wheel C^2 , immediately to the left of B^2 , is fixed a wheel, called in mechanics a ratchet wheel, which is driven by a claw which constantly rests in its teeth. This claw is in such a position as to permit the wheel C^2 to move in obedience to the action of the adding bolt, but to resist its motion in the contrary direction. It is drawn back by a spiral spring, but its recoil is prevented by a hook which sustains it; which hook, however, is capable of being withdrawn, and when withdrawn, the aforesaid spiral spring would draw back the claw, and make it fall through one tooth of the ratchet wheel. Now, at the moment that the division between 9 and 0 on the dial B^2 passes under the index, a thumb placed on the axis of this dial touches a trigger which raises out of the notch the hook which sustains the claw just mentioned, and allows it to fall back by the recoil of the spring, and to drop into the next tooth of the ratchet wheel. This process, however, produces no immediate effect upon the position of the wheel C^2 , and is merely preparatory to an action intended to take place during the second quarter of a turn of the moving axis. It is in effect a memorandum taken by the machine of a carriage to be made in the next quarter of a turn.

During the second quarter of a turn, a finger placed on the axis of the dial B^2 is made to revolve, and it encounters the heel of the above-mentioned claw. As it moves forward it drives the claw before it: and this claw, resting in the teeth of the ratchet wheel fixed upon the axis of the dial C^2 drives forward that wheel, and with it the dial. But the length and position of the finger which drives the claw limits its action, so as to move the claw forward through such a space only as will cause the dial C^2 to advance through a single division; at which point it is again caught and

retained by the hook. This will be added to the number under its index, and the requisite carriage from B^2 to C^2 will be accomplished.

In connexion with every dial is placed a similar ratchet wheel with a similar claw, drawn by a similar spring, sustained by a similar hook, and acted upon by a similar thumb and trigger; and therefore the necessary carriages, throughout the whole machinery, take place in the same manner and by similar means.

During the second quarter of a turn, such of the carrying claws as have been allowed to recoil in the first, third, and fifth rows, are drawn up by the fingers on the axes of the adjacent dials; and, during the fourth quarter of a turn, such of the carrying claws on the second and fourth rows as have been allowed to recoil during the third quarter of a turn, are in like manner drawn up by the carrying fingers on the axes of the adjacent dials. It appears that the carriages proceed alternately from right to left along the horizontal rows during the second and fourth quarters of a turn; in the one, they pass along the first, third, and fifth rows, and in the other, along the second and fourth.

There are two systems of waves of mechanical action continually flowing from the bottom to the top; and two streams of similar action constantly passing from the right to the left. The crests of the first system of adding waves fall upon the last difference, and upon every alternate one proceeding upwards; while the crests of the other system touch upon the intermediate differences. The first stream of carrying action passes from right to left along the highest row and every alternate row, while the second stream passes along the intermediate rows.

Such is a very rapid and general outline of this machinery. Its wonders, however, are still greater in its details than even in its broader features. Although we despair of doing it justice by any description which can be attempted here, yet we should not fulfil the duty we owe to our readers, if we did not call their attention at least to a few of the instances of consummate skill which are scattered, with a prodigality characteristic of the highest order of inventive genius, throughout this astonishing mechanism.

In the general description which we have given of the mechanism for *carrying*, it will be observed, that the preparation for every carriage is stated to be made during the previous addition, by the disengagement of the

carrying claw before mentioned, and by its consequent recoil, urged by the spiral spring with which it is connected; but it may, and does, frequently happen, that though the process of addition may not have rendered a carriage necessary, one carriage may itself produce the necessity for another. This is a contingency not provided against in the mechanism as we have described it: the case would occur in the scheme represented in [fig. 1](#), if the figure under the index of C^2 were 4 instead of 3. The addition of the number 5 at the index of C^3 would, in this case, in the first quarter of a turn, bring 9 to the index of C^2 : this would obviously render no carriage necessary, and of course no preparation would be made for one by the mechanism—that is to say, the carrying claw of the wheel D^2 would not be detached. Meanwhile a carriage upon C^2 has been rendered necessary by the addition made in the first quarter of a turn to B^2 . This carriage takes place in the ordinary way, and would cause the dial C^2 , in the second quarter of a turn, to advance from 9 to 0: this would make the necessary preparation for a carriage from C^2 to D^2 . But unless some special arrangement was made for the purpose, that carriage would not take place during the second quarter of a turn. This peculiar contingency is provided against by an arrangement of singular mechanical beauty, and which, at the same time, answers another purpose—that of equalizing the resistance opposed to the moving power by the carrying mechanism. The fingers placed on the axes of the several dials in the row D^2 , do not act at the same instant on the carrying claws adjacent to them; but they are so placed, that their action may be distributed throughout the second quarter of a turn in regular succession. Thus the finger on the axis of the dial A^2 first encounters the claw upon B^2 , and drives it through one tooth immediately forwards; the finger on the axis of B^2 encounters the claw upon C^2 and drives it through one tooth; the action of the finger on C^2 on the claw on D^2 next succeeds, and so on. Thus, while the finger on B^2 acts on C^2 , and causes the division from 9 to 0 to pass under the index, the thumb on C^2 at the same instant acts on the trigger, and detaches the carrying claw on D^2 , which is forthwith encountered by the carrying finger on C^2 , and driven forward one tooth. The dial D^2 accordingly moves forward one division, and 5 is brought under the index. This arrangement is beautifully effected

by placing the several fingers, which act upon the carrying claws, *spirally* on their axes, so that they come into action in regular succession.

We have stated that, at the commencement of each revolution of the moving axis, the bolts which drive the dials of the first, third, and fifth rows, are shot. The process of shooting these bolts must therefore have taken place during the last quarter of the preceding revolution; but it is during that quarter of a turn that the carriages are effected in the second and fourth rows. Since the bolts which drive the dials of the first, third, and fifth rows, have no mechanical connexion with the dials in the second and fourth rows, there is nothing in the process of shooting those bolts incompatible with that of moving the dials of the second and fourth rows: hence these two processes may both take place during the same quarter of a turn. But in order to equalize the resistance to the moving power, the same expedient is here adopted as that already described in the process of carrying. The arms which shoot the bolts of each row of dials are arranged spirally, so as to act successively throughout the quarter of a turn. There is, however, a contingency which, under certain circumstances, would here produce a difficulty which must be provided against. It is possible, and in fact does sometimes happen, that the process of carrying causes a dial to move under the index from 0 to 1. In that case, the bolt, preparatory to the next addition, ought not to be shot until after the carriage takes place; for if the arm which shoots it passes its point of action before the carriage takes place, the bolt will be moved out of its sphere of action, and will not be shot, which, as we have already explained, must always happen when 0 is at the index: therefore no addition would in this case take place during the next quarter of a turn of the axis; whereas, since 1 is brought to the index by the carriage, which immediately succeeds the passage of the arm which ought to bolt, 1 should be added during the next quarter of a turn. It is plain, accordingly, that the mechanism should be so arranged, that the action of the arms, which shoot the bolts successively, should immediately follow the action of those fingers which raise the carrying claws successively; and therefore either a separate quarter of a turn should be appropriated to each of those movements, or if they be executed in the same quarter of a turn, the mechanism must be so constructed, that the arms which shoot the bolts successively, shall severally follow immediately after those which raise the carrying claws successively. The latter object is attained by a mechanical arrangement of singular felicity, and partaking of that elegance which

characterises all the details of this mechanism. Both sets of arms are spirally arranged on their respective axes, so as to be carried through their period in the same quarter of a turn; but the one spiral is shifted a few degrees, in angular position, behind the other, so that each pair of corresponding arms succeed each other in the most regular order,—equalizing the resistance, economizing time, harmonizing the mechanism, and giving to the whole mechanical action the utmost practical perfection.

The system of mechanical contrivances by which the results, here attempted to be described, are attained, form only one order of expedients adopted in this machinery;—although such is the perfection of their action, that in any ordinary case they would be regarded as having attained the ends in view with an almost superfluous degree of precision. Considering, however, the immense importance of the purposes which the mechanism was destined to fulfil, its inventor determined that a higher order of expedients should be superinduced upon those already described; the purpose of which should be to obliterate all small errors or inequalities which might, even by remote possibility, arise, either from defects in the original formation of the mechanism, from inequality of wear, from casual strain or derangement,—or, in short, from any other cause whatever. Thus the movements of the first and principal parts of the mechanism were regarded by him merely as a first, though extremely nice approximation, upon which a system of small corrections was to be subsequently made by suitable and independent mechanism. This supplementary system of mechanism is so contrived, that if one or more of the moving parts of the mechanism of the first order be slightly out of their places, they will be forced to their exact position by the action of the mechanical expedients of the second order to which we now allude. If a more considerable derangement were produced by any accidental disturbance, the consequence would be that the supplementary mechanism would cause the whole system to become locked, so that not a wheel would be capable of moving; the impelling power would necessarily lose all its energy, and the machine would stop. The consequence of this exquisite arrangement is, that the machine will either calculate rightly, or not at all.

The supernumerary contrivances which we now allude to, being in a great degree unconnected with each other, and scattered through the machinery to a certain extent, independent of the mechanical arrangement

of the principal parts, we find it difficult to convey any distinct notion of their nature or form.

In some instances they consist of a roller resting between certain curved surfaces, which has but one position of stable equilibrium, and that position the same, however the roller or the curved surfaces may wear. A slight error in the motion of the principal parts would make this roller for the moment rest on one of the curves; but, being constantly urged by a spring, it would press on the curved surface in such a manner as to force the moving piece on which that curved surface is formed, into such a position that the roller may rest between the two surfaces; that position being the one which the mechanism should have. A greater derangement would bring the roller to the crest of the curve, on which it would rest in instable equilibrium; and the machine would either become locked, or the roller would throw it as before into its true position.

In other instances a similar object is attained by a solid cone being pressed into a conical seat; the position of the axis of the cone and that of its seat being necessarily invariable, however the cone may wear: and the action of the cone upon the seat being such, that it cannot rest in any position except that in which the axis of the cone coincides with the axis of its seat.

Having thus attempted to convey a notion, however inadequate, of the calculating section of the machinery, we shall proceed to offer some explanation of the means whereby it is enabled, to print its calculations in such a manner as to preclude the possibility of error in any individual printed copy.

On the axle of each of the wheels which express the calculated number of the table T, there is fixed a solid piece of metal, formed into a curve, not unlike the wheel in a common clock, which is called the *snail*. This curved surface acts against the arm of a lever, so as to raise that arm to a higher or lower point according to the position of the dial with which the snail is connected. Without entering into a more minute description, it will be easily understood that the snail may be so formed that the arm of the lever shall be raised to ten different elevations, corresponding to the ten figures of the dial which may be brought under the index. The opposite arm of the lever here described puts in motion a solid arch, or sector, which carries ten punches:

each punch bearing on its face a raised character of a figure, and the ten punches bearing the ten characters, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0. It will be apparent from what has been just stated, that this *type sector* (as it is called) will receive ten different attitudes, corresponding to the ten figures which may successively be brought under the index of the dial-plate. At a point over which the type sector is thus moved, and immediately under a point through which it plays, is placed a frame, in which is fixed a plate of copper. Immediately over a certain point through which the type sector moves, is likewise placed a *bent lever*, which, being straightened, is forcibly pressed upon the punch which has been brought under it. If the type sector be moved, so as to bring under the bent lever one of the steel punches above mentioned, and be held in that position for a certain time, the bent lever, being straightened, acts upon the steel punch, and drives it against the face of the copper beneath, and thus causes a sunken impression of the character upon the punch to be left upon the copper. If the copper be now shifted slightly in its position, and the type sector be also shifted so as to bring another punch under the bent lever, another character may be engraved on the copper by straightening the bent lever, and pressing it on the punch as before. It will be evident, that if the copper was shifted from right to left through a space equal to two figures of a number, and, at the same time, the type sector so shifted as to bring the punches corresponding to the figures of the number successively under the bent lever, an engraved impression of the number might thus be obtained upon the copper by the continued action of the bent lever. If, when one line of figures is thus obtained, a provision be made to shift the copper in a direction at right angles to its former motion, through a space equal to the distance between two lines of figures, and at the same time to shift it through a space in the other direction equal to the length of an entire line, it will be evident that another line of figures might be printed below the first in the same manner.

The motion of the type sector, here described, is accomplished by the action of the snail upon the lever already mentioned. In the case where the number calculated is that expressed in [fig. 1](#), the process would be as follows:—The snail of the wheel F^1 , acting upon the lever, would throw the type sector into such an attitude, that the punch bearing the character 0 would come under the bent lever. The next turn of the moving axis would cause the bent lever to press on the tail of the punch, and the character 0 would be impressed upon the copper. The bent lever being again drawn up,

the punch would recoil from the copper by the action of a spring; the next turn of the moving axis would shift the copper through the interval between two figures, so as to bring the point destined to be impressed with the next figure under the bent lever. At the same time, the snail of the wheel E would cause the type sector to be thrown into the same attitude as before, and the punch would be brought under the bent lever; the next turn would impress the figure beside the former one, as before described. The snail upon the wheel D would now come into action, and throw the type sector into that position in which the punch bearing the character 7 would come under the bent lever, and at the same time the copper would be shifted through the interval between two figures; the straightening of the lever would next follow, and the character 7 would be engraved. In the same manner, the wheels C, B, and A would successively act by means of their snails; and the copper being shifted, and the lever allowed to act, the number 007776 would be finally engraved upon the copper: this being accomplished, the calculating machinery would next be called into action, and another calculation would be made, producing the next number of the Table exhibited in [fig. 5](#). During this process the machinery would be engaged in shifting the copper both in the direction of its length and its breadth, with a view to commence the printing of another line; and this change of position would be accomplished at the moment when the next calculation would be completed: the printing of the next number would go on like the former, and the operation of the machine would proceed in the same manner, calculating and printing alternately. It is not, however, at all necessary—though we have here supposed it, for the sake of simplifying the explanation—that the calculating part of the mechanism should have its action suspended while the printing part is in operation, or *vice versa*; it is not intended, in fact, to be so suspended in the actual machinery. The same turn of the axis by which one number is printed, executes a part of the movements necessary for the succeeding calculation; so that the whole mechanism will be simultaneously and continuously in action.

Of the mechanism by which the position of the copper is shifted from figure to figure, from line to line, we shall not attempt any description. We feel that it would be quite vain. Complicated and difficult to describe as every other part of this machinery is, the mechanism for moving the copper is such as it would be quite impossible to render at all intelligible, without numerous illustrative drawings.

The engraved plate of copper obtained in the manner above described, is designed to be used as a mould from which a stereotyped plate may be cast; or, if deemed advisable, it may be used as the immediate means of printing. In the one case we should produce a table, printed from type, in the same manner as common letter-press printing; in the other an engraved table. If it be thought most advisable to print from the stereotyped plates, then as many stereotyped plates as may be required may be taken from the copper mould; so that when once a table has been calculated and engraved by the machinery, the whole world may be supplied with stereotyped plates to print it, and may continue to be so supplied for an unlimited period of time. There is no practical limit to the number of stereotyped plates which may be taken from the engraved copper; and there is scarcely any limit to the number of printed copies which may be taken from any single stereotyped plate. Not only, therefore, is the numerical table by these means engraved and stereotyped with infallible accuracy, but such stereotyped plates are producible in unbounded quantity. Each plate, when produced, becomes itself the means of producing printed copies of the table, in accuracy perfect, and in number without limit.

Unlike all other machinery, the calculating mechanism produces, not the object of consumption, but the machinery by which that object may be made. To say that it computes and prints with infallible accuracy, is to understate its merits:—it computes and fabricates *the means* of printing with absolute correctness and in unlimited abundance.

For the sake of clearness, and to render ourselves more easily intelligible to the general reader, we have in the preceding explanation thrown the mechanism into an arrangement somewhat different from that which is really adopted. The dials expressing the numbers of the tables of the successive differences are not placed, as we have supposed them, in horizontal rows, and read from right to left, in the ordinary way; they are, on the contrary, placed vertically, one below the other, and read from top to bottom. The number of the table occupies the first vertical column on the right, the units being expressed on the lowest dial, and the tens on the next above that, and so on. The first difference occupies the next vertical column on the left; and the numbers of the succeeding differences occupy vertical columns, proceeding regularly to the left; the constant difference being on the last vertical column. It is intended in the machine now in progress to

introduce six orders of differences, so that there will be seven columns of dials; it is also intended that the calculations shall extend to eighteen places of figures: thus each column will have eighteen dials. We have referred to the dials as if they were inscribed upon the faces of wheels, whose axes are horizontal and planes vertical. In the actual machinery the axes are vertical and the planes horizontal, so that the edges of the *figure wheels*, as they are called, are presented to the eye. The figures are inscribed, not upon the dial-plate, but around the surface of a small cylinder or barrel, placed upon the axis of the figure wheel, which revolves with it; so that as the figure wheel revolves, the figures on the barrel are successively brought to the front, and pass under an index engraved upon a plate of metal immediately above the barrel. This arrangement has the obvious practical advantage, that, instead of each figure wheel having a separate axis, all the figure wheels of the same vertical column revolve on the same axis; and the same observation will apply to all the wheels with which the figure wheels are in mechanical connexion. This arrangement has the further mechanical advantage over that which has been assumed for the purposes of explanation, that the friction of the wheel-work on the axes is less in amount, and more uniformly distributed, than it could be if the axes were placed in the horizontal position.

A notion may therefore be formed of the front elevation of the calculating part of the mechanism, by conceiving seven steel axes erected, one beside another, on each of which shall be placed eighteen wheels,^[12] five inches in diameter, having cylinders or barrels upon them an inch and a half in height, and inscribed, as already stated, with the ten arithmetical characters. The entire elevation of the machinery would occupy a space measuring ten feet broad, ten feet high, and five feet deep. The process of calculation would be observed by the alternate motion of the figure wheels on the several axes. During the first quarter of a turn, the wheels on the first, third, and fifth axes would turn, receiving their addition from the second, fourth, and sixth; during the second quarter of a turn, such of the wheels on the first, third, and fifth axes, to which carriages are due, would be moved forward one additional figure; the second, fourth, and sixth columns of wheels being all this time quiescent. During the third quarter of a turn, the second, fourth, and sixth columns would be observed to move, receiving their additions from the third, fifth, and seventh axes; and during the fourth quarter of a turn, such of these wheels to which carriages are due,

would be observed to move forward one additional figure; the wheels of the first, third, and fifth columns being quiescent during this time.

[12]The wheels, and every other part of the mechanism except the axes, springs, and such parts as are necessarily of steel, are formed of an alloy of copper with a small portion of tin.

It will be observed that the wheels of the seventh column are always quiescent in this process; and it may be asked, of what use they are, and whether some mechanism of a fixed nature would not serve the same purpose? It must, however, be remembered, that for different tables there will be different constant differences; and that when the calculation of a table is about to commence, the wheels on the seventh axis must be moved by the hand, so as to express the constant difference, whatever it may be. In tables, also, which have not a difference rigorously constant, it will be necessary, after a certain number of calculations, to change the constant difference by the hand; and in this case the wheels of the seventh axis must be moved when occasion requires. Such adjustment, however, will only be necessary at very distant intervals, and after a considerable extent of printing and calculation has taken place; and when it is necessary, a provision is made in the machinery by which notice will be given by the sounding of a bell, so that the machine may not run beyond the extent of its powers of calculation.

Immediately behind the seven axes on which the figure wheels revolve, are seven other axes; on which are placed, first, the wheels already described as driven by the figure wheels, and which bear upon them the wedge which withdraws the bolt immediately over these latter wheels, and on the same axis is placed the adding bolt. From the bottom of this bolt there projects downwards the pin, which acts upon the unbolting wedge by which the bolt is withdrawn: from the upper surface of the bolt proceeds a tooth, which, when the bolt is shot, enters between the teeth of the adding wheel, which turns on the same axis, and is placed immediately above the bolt: its teeth, on which the bolt acts, are like the teeth of a crown wheel, and are presented downwards. The bolt is fixed upon this axis, and turns with it; but the adding wheel above the bolt, and the unbolting wheel below it, both turn upon the axis, and independently of it. When the axis is made to revolve by the moving power, the bolt revolves with it; and so long as the

tooth of the bolt remains inserted between those of the adding wheel, the latter is likewise moved; but when the lower pin of the bolt encounters the unbolting wedge on the lower wheel, the tooth of the bolt is withdrawn, and the motion of the adding wheel is stopped. This adding wheel is furnished with spur teeth, besides the crown teeth just mentioned; and these spur teeth are engaged with those of that unbolting wheel which is in connexion with the adjacent figure wheel to which the addition is to be made. By such an arrangement it is evident that the revolution of the bolt will necessarily add to the adjacent figure wheel the requisite number.

It will be perceived, that upon the same axis are placed an unbolting wheel, a bolt, and an adding wheel, one above the other, for every figure wheel; and as there are eighteen figure wheels there will be eighteen tiers; each tier formed of an unbolting wheel, a bolt, and an adding wheel, placed one above the other; the wheels on this axis all revolving independent of the axis, but the bolts being all fixed upon it. The same observations, of course, will apply to each of the seven axes.

At the commencement of every revolution of the adding axes, it is evident that the several bolts placed upon them must be shot in order to perform the various additions. This is accomplished by a third set of seven axes, placed at some distance behind the range of the wheels, which turn upon the adding axes: these are called *bolting axes*. On these bolting axes are fixed, so as to revolve with them, a bolting finger opposite to each bolt; as the bolting axis is made to revolve by the moving power, the bolting finger is turned, and as it passes near the bolt, it encounters the shoulder of a hammer or lever, which strikes the heel of the bolt, and presses it forward so as to shoot its tooth between the crown teeth of the adding wheel. The only exception to this action is the case in which 0 happens to be at the index of the figure wheel; in that case, the lever or hammer, which the bolting finger would encounter, is, as before stated, lifted out of the way of the bolting finger, so that it revolves without encountering it. It is on the bolting axes that the fingers are spirally arranged so as to equalize their action, as already explained.

The same axes in the front of the machinery on which the figure wheels turn, are made to serve the purpose of *carrying*. Each of these bear a series of fingers which turn with them, and which encounter a carrying claw,

already described, so as to make the carriage: these carrying fingers are also spirally arranged on their axes, as already described.

Although the absolute accuracy which appears to be ensured by the mechanical arrangements here described is such as to render further precautions nearly superfluous, still it may be right to state, that, supposing it were possible for an error to be produced in calculation, this error could be easily and speedily detected in the printed tables: it would only be necessary to calculate a number of the table taken at intervals, through which the mechanical action of the machine has not been suspended, and during which it has received no adjustment by the hand: if the computed number be found to agree with those printed, it may be taken for granted that all the intermediate numbers are correct; because, from the nature of the mechanism, and the principle of computation, an error occurring in any single number of the table would be unavoidably entailed, in an increasing ratio, upon all the succeeding numbers.

We have hitherto spoken merely of the practicability of executing by the machinery, when completed, that which its inventor originally contemplated—namely, the calculating and printing of all numerical tables, derived by the method of differences from a constant difference. It has, however, happened that the actual powers of the machinery greatly transcend those contemplated in its original design:—they not only have exceeded the most sanguine anticipations of its inventor, but they appear to have an extent to which it is utterly impossible, even for the most acute mathematical thinker, to fix a probable limit. Certain subsidiary mechanical inventions have, in the progress of the enterprise, been, by the very nature of the machinery, suggested to the mind of the inventor, which confer upon it capabilities which he had never foreseen. It would be impossible even to enumerate, within the limits of this article, much less to describe in detail, those extraordinary mechanical arrangements, the effects of which have not failed to strike with astonishment every one who has been favoured with an opportunity of witnessing them, and who has been enabled, by sufficient mathematical attainments, in any degree to estimate their probable consequences.

As we have described the mechanism, the axes containing the several differences are successively and regularly added one to another; but there are certain mechanical adjustments, and these of a very simple nature,

which being thrown into action, will cause a difference of any order to be added any number of times to a difference of any other order; and that either proceeding backwards or forwards, from a difference of an inferior to one of a superior order, and *vice versa*.^[13]

^[13]The machine was constructed with the intention of tabulating the equation $\Delta^7 u = 0$, but, by the means above alluded to, it is capable of tabulating such equations as the following: $\Delta^7 u = a \Delta u$, $\Delta^7 u = a \Delta^3 u$, $\Delta^7 u = \text{units figure of } \Delta u$.

Among other peculiar mechanical provisions in the machinery is one by which, when the table for any order of difference amounts to a certain number, a certain arithmetical change would be made in the constant difference. In this way a series may be tabulated by the machine, in which the constant difference is subject to periodical change; or the very nature of the table itself may be subject to periodical change, and yet to one which has a regular law.

Some of these subsidiary powers are peculiarly applicable to calculations required in astronomy, and are therefore of eminent and immediate practical utility: others there are by which tables are produced, following the most extraordinary, and apparently capricious, but still regular laws. Thus a table will be computed, which, to any required extent, shall coincide with a given table, and which shall deviate from that table for a single term, or for any required number of terms, and then resume its course, or which shall permanently alter the law of its construction. Thus the engine has calculated a table which agreed precisely with a table of square numbers, until it attained the hundred and first term, which was not the square of 101, nor were any of the subsequent numbers squares. Again, it has computed a table which coincided with the series of natural numbers, as far as 100,000,001, but which subsequently followed another law. This result was obtained, not by working the engine through the whole of the first table, for that would have required an enormous length of time; but by showing, from the arrangement of the mechanism, that it must continue to exhibit the succession of natural numbers, until it would reach 100,000,000. To save time, the engine was set by the hand to the number 99999995, and was then put in regular operation. It produced successively the following numbers.

^[14]

99,999,996
99,999,997
99,999,998
99,999,999
100,000,000
100,010,002
100,030,003
100,060,004
100,100,005
100,150,006
&c. &c.

[14] Such results as this suggest a train of reflection on the nature and operation of general laws, which would lead to very curious and interesting speculations. The natural philosopher and astronomer will be hardly less struck with them than the metaphysician and theologian.

Equations have been already tabulated by the portion of the machinery which has been put together, which are so far beyond the reach of the present power of mathematics, that no distant term of the table can be predicted, nor any function discovered capable of expressing its general law. Yet the very fact of the table being produced by mechanism of an invariable form, and including a distinct principle of mechanical action, renders it quite manifest that *some* general law must exist in every table which it produces. But we must dismiss these speculations: we feel it impossible to stretch the powers of our own mind, so as to grasp the probable capabilities of this splendid production of combined mechanical and mathematical genius; much less can we hope to enable others to appreciate them, without being furnished with such means of comprehending them as those with which we have been favoured. Years must in fact elapse, and many enquirers direct their energies to the cultivation of the vast field of research thus opened, before we can fully estimate the extent of this triumph of matter over mind. 'Nor is it,' says Mr Colebrooke, 'among the least curious results of this ingenious device, that it affords a new opening for discovery, since it is applicable, as has been shown by its inventor, to surmount novel difficulties of analysis. Not confined to constant differences, it is available in every case of differences that follow a definite law, reducible therefore to an equation. An engine adjusted to the purpose being set to work, will produce any distant term, or

succession of terms, required—thus presenting the numerical solution of a problem, even though the analytical solution be yet undetermined.' That the future path of some important branches of mathematical enquiry must now in some measure be directed by the dictates of mechanism, is sufficiently evident; for who would toil on in any course of analytical enquiry, in which he must ultimately depend on the expensive and fallible aid of human arithmetic, with an instrument in his hands, in which all the dull monotony of numerical computation is turned over to the untiring action and unerring certainty of mechanical agency?

It is worth notice, that each of the axes in front of the machinery on which the figure wheels revolve, is connected with a bell, the tongue of which is governed by a system of levers, moved by the several figure wheels; an adjustment is provided by which the levers shall be dismissed, so as to allow the hammer to strike against the bell, whenever any proposed number shall be exhibited on the axis. This contrivance enables the machine to give notice to its attendants at any time that an adjustment may be required.

Among a great variety of curious accidental properties (so to speak) which the machine is found to possess, is one by which it is capable of solving numerical equations which have rational roots. Such an equation being reduced (as it always may be) by suitable transformations to that state in which the roots shall be whole numbers, the values 0, 1, 2, 3, &c., are substituted for the unknown quantity, and the corresponding values of the equation ascertained. From these a sufficient number of differences being derived, they are set upon the machine. The machine being then put in motion, the table axis will exhibit the successive values of the formula, corresponding to the substitutions of the successive whole numbers for the unknown quantity: at length the number exhibited on the table axis will be 0, which will evidently correspond to a root of the equation. By previous adjustment, the bell of the table axis will in this case ring and give notice of the exhibition of the value of the root in another part of the machinery.

If the equation have imaginary roots, the formula being necessarily a maximum or minimum on the occurrence of such roots, the first difference will become nothing; and the dials of that axis will under such circumstances present to the respective indices. By previous adjustment, the bell of this axis would here give notice of a pair of imaginary roots.

Mr Colebrooke speculates on the probable extension of these powers of the machine: 'It may not therefore be deemed too sanguine an anticipation when I express the hope that an instrument which, in its simpler form, attains to the extraction of roots of numbers, and approximates to the roots of equations, may, in a more advanced state of improvement, rise to the approximate solution of algebraic equations of elevated degrees. I refer to solutions of such equations proposed by La Grange, and more recently by other annalists, which involve operations too tedious and intricate for use, and which must remain without efficacy, unless some mode be devised of abridging the labour, or facilitating the means of its performance. In any case this engine tends to lighten the excessive and accumulating burden of arithmetical application of mathematical formulæ, and to relieve the progress of science from what is justly termed by the author of this invention, the overwhelming encumbrance of numerical detail.'

Although there are not more than eighteen figure wheels on each axis, and therefore it might be supposed that the machinery was capable of calculating only to the extent of eighteen decimal places; yet there are contrivances connected with it, by which, in two successive calculations, it will be possible to calculate even to the extent of thirty decimal places. Its powers, therefore, in this respect, greatly exceed any which can be required in practical science. It is also remarkable, that the machinery is capable of producing the calculated results *true to the last figure*. We have already explained, that when the figure which would follow the last is greater than 4, then it would be necessary to increase the last figure by 1; since the excess of the calculated number above the true value would in such case be less than its defect from it would be, had the regularly computed final figure been adopted: this is a precaution necessary in all numerical tables, and it is one which would hardly have been expected to be provided for in the calculating machinery.

As might be expected in a mechanical undertaking of such complexity and novelty, many practical difficulties have since its commencement been encountered and surmounted. It might have been foreseen, that many expedients would be adopted and carried into effect, which farther experiments would render it necessary to reject; and thus a large source of additional expense could scarcely fail to be produced. To a certain extent this has taken place; but owing to the admirable system of mechanical

drawings, which in every instance Mr Babbage has caused to be made, and owing to his own profound acquaintance with the practical working of the most complicated mechanism, he has been able to predict in every case what the result of any contrivance would be, as perfectly from the drawing, as if it had been reduced to the form of a working model. The drawings, consequently, form a most extensive and essential part of the enterprise. They are executed with extraordinary ability and precision, and may be considered as perhaps the best specimens of mechanical drawings which have ever been executed. It has been on these, and on these only, that the work of invention has been bestowed. In these, all those progressive modifications suggested by consideration and study have been made; and it was not until the inventor was fully satisfied with the result of any contrivance, that he had it reduced to a working form. The whole of the loss which has been incurred by the necessarily progressive course of invention, has been the expense of rejected drawings. Nothing can perhaps more forcibly illustrate the extent of labour and thought which has been incurred in the production of this machinery, than the contemplation of the working drawings which have been executed previously to its construction: these drawings cover above a thousand square feet of surface, and many of them are of the most elaborate and complicated description.

One of the practical difficulties which presented themselves at a very early stage in the progress of this undertaking, was the impossibility of bearing in mind all the variety of motions propagated simultaneously through so many complicated trains of mechanism. Nothing but the utmost imaginable harmony and order among such a number of movements, could prevent obstructions arising from incompatible motions encountering each other. It was very soon found impossible, by a mere act of memory, to guard against such an occurrence; and Mr Babbage found, that, without some effective expedient by which he could at a glance see what every moving piece in the machinery was doing at each instant of time, such inconsistencies and obstructions as are here alluded to must continually have occurred. This difficulty was removed by another invention of even a more general nature than the calculating machinery itself, and pregnant with results probably of higher importance. This invention consisted in the contrivance of a scheme of *mechanical notation* which is generally applicable to all machinery whatsoever; and which is exhibited on a table or plan consisting of two distinct sections. In the first is traced, by a peculiar

system of signs, the origin of every motion which takes place throughout the machinery; so that the mechanist or inventor is able, by moving his finger along a certain line, to follow out the motion of every piece from effect to cause, until he arrives at the prime mover. The same sign which thus indicates the *source* of motion indicates likewise the *species* of motion, whether it be continuous or reciprocating, circular or progressive, &c. The same system of signs further indicates the nature of the mechanical connexion between the mover and the thing moved, whether it be permanent and invariable (as between the two arms of a lever), or whether the mover and the moved are separate and independent pieces, as is the case when a pinion drives a wheel; also whether the motion of one piece necessarily implies the motion of another; or when such motion in the one is interrupted, and in the other continuous, &c.

The second section of the table divides the time of a complete period of the machinery into any required number of parts; and it exhibits in a map, as it were, that which every part of the machine is doing at each moment of time. In this way, incompatibility in the motions of different parts is rendered perceptible at a glance. By such means the contriver of machinery is not merely prevented from introducing into one part of the mechanism any movement inconsistent with the simultaneous action of the other parts; but when he finds that the introduction of any particular movement is necessary for his purpose, he can easily and rapidly examine the whole range of the machinery during one of its periods, and can find by inspection whether there is any, and what portion of time, at which no motion exists incompatible with the desired one, and thus discover a *niche*, as it were, in which to place the required movement. A further and collateral advantage consists in placing it in the power of the contriver to exercise the utmost possible economy of *time* in the application of his moving power. For example, without some instrument of mechanical enquiry equally powerful with that now described, it would be scarcely possible, at least in the first instance, so to arrange the various movements that they should be all executed in the least possible number of revolutions of the moving axis. Additional revolutions would almost inevitably be made for the purpose of producing movements and changes which it would be possible to introduce in some of the phases of previous revolutions: and there is no one acquainted with the history of mechanical invention who must not be aware, that in the progressive contrivance of almost every machine the

earliest arrangements are invariably defective in this respect; and that it is only by a succession of improvements, suggested by long experience, that that arrangement is at length arrived at, which accomplishes all the necessary motions in the shortest possible time. By the application of the mechanical notation, however, absolute perfection may be arrived at in this respect; even before a single part of the machinery is constructed, and before it has any other existence than that which it obtains upon paper.

Examples of this class of advantages derivable from the notation will occur to the mind of every one acquainted with the history of mechanical invention. In the common suction-pump, for example, the effective agency of the power is suspended during the descent of the piston. A very simple contrivance, however, will transfer to the descent the work to be accomplished in the next ascent; so that the duty of four strokes of the piston may thus be executed in the time of two. In the earlier applications of the steam-engine, that machine was applied almost exclusively to the process of pumping; and the power acted only during the descent of the piston, being suspended during its ascent. When, however, the notion of applying the engine to the general purposes of manufacture occurred to the mind of Watt, he saw that it would be necessary to cause it to produce a continued rotatory motion; and, therefore, that the intervals of intermission must be filled up by the action of the power. He first proposed to accomplish this by a second cylinder working alternately with the first; but it soon became apparent that the blank which existed during the upstroke in the action of the power, might be filled up by introducing the steam at both ends of the cylinder alternately. Had Watt placed before him a scheme of mechanical notation such as we allude to, this expedient would have been so obtruded upon him that he must have adopted it from the first.

One of the circumstances from which the mechanical notation derives a great portion of its power as an instrument of investigation and discovery, is that it enables the inventor to dismiss from his thoughts, and to disencumber his imagination of the arrangement and connexion of the mechanism; which, when it is very complex (and it is in that case that the notation is most useful), can only be kept before the mind by an embarrassing and painful effort. In this respect the powers of the notation may not inaptly be illustrated by the facilities derived in complex and difficult arithmetical questions from the use of the language and notation of algebra. When once

the peculiar conditions of the question are translated into algebraical signs, and 'reduced to an equation,' the computist dismisses from his thoughts all the circumstances of the question, and is relieved from the consideration of the complicated relations of the quantities of various kinds which may have entered it. He deals with the algebraical symbols, which are the representatives of those quantities and relations, according to certain technical rules of a general nature, the truth of which he has previously established; and, by a process almost mechanical, he arrives at the required result. What algebra is to arithmetic, the notation we now allude to is to mechanism. The various parts of the machinery under consideration being once expressed upon paper by proper symbols, the enquirer dismisses altogether from his thoughts the mechanism itself, and attends only to the symbols; the management of which is so extremely simple and obvious, that the most unpractised person, having once acquired an acquaintance with the signs, cannot fail to comprehend their use.

A remarkable instance of the power and utility of this notation occurred in a certain stage of the invention of the calculating machinery. A question arose as to the best method of producing and arranging a certain series of motions necessary to print and calculate a number. The inventor, assisted by a practical engineer of considerable experience and skill, had so arranged these motions, that the whole might be performed by twelve revolutions of the principal moving axis. It seemed, however, desirable, if possible, to execute these motions by a less number of revolutions. To accomplish this, the engineer sat down to study the complicated details of a part of the machinery which had been put together; the inventor at the same time applied himself to the consideration of the arrangement and connexion of the symbols in his scheme of notation. After a short time, by some transposition of symbols, he caused the received motions to be completed by eight turns of the axis. This he accomplished by transferring the symbols which occupied the last four divisions of his scheme, into such blank spaces as he could discover in the first eight divisions; due care being taken that no symbols should express actions at once simultaneous and incompatible. Pushing his enquiry, however, still further, he proceeded to ascertain whether his scheme of symbols did not admit of a still more compact arrangement, and whether eight revolutions were not more than enough to accomplish what was required. Here the powers of the practical engineer completely broke down. By no effort could he bring before his mind such a

view of the complicated mechanism as would enable him to decide upon any improved arrangement. The inventor, however, without any extraordinary mental exertion, and merely by sliding a bit of ruled pasteboard up and down his plan, in search of a vacancy where the different motions might be placed, at length contrived to pack all the motions, which had previously occupied eight turns of the handle, into five turns. The symbolic instrument with which he conducted the investigation, now informed him of the impossibility of reducing the action of the machine to a more condensed form. This appeared by the fulness of every space along the lines of compatible action. It was, however, still possible, by going back to the actual machinery, to ascertain whether movements, which, under existing arrangements, were incompatible, might not be brought into harmony. This he accordingly did, and succeeded in diminishing the number of incompatible conditions, and thereby rendered it possible to make actions simultaneous which were before necessarily successive. The notation was now again called into requisition, and a new disposition of the parts was made. At this point of the investigation, this extraordinary instrument of mechanical analysis put forth one of its most singular exertions of power. It presented to the eye of the engineer two currents of mechanical action, which, from their nature, could not be simultaneous; and each of which occupied a complete revolution of the axis, except about a twentieth; the one occupying the last nineteen-twentieths of a complete revolution of the axis, and the other occupying the first nineteen-twentieths of a complete revolution. One of these streams of action was, the successive picking up by the carrying fingers of the successive carrying claws; and the other was, the successive shooting of nineteen bolts by the nineteen bolting fingers. The notation rendered it obvious, that as the bolting action commenced a small space below the commencement of the carrying, and ended an equal space below the termination of the carrying, the two streams of action could be made to flow after one another in one and the same revolution of the axis. He thus succeeded in reducing the period of completing the action to four turns of the axis; when the notation again informed him that he had again attained a limit of condensed action, which could not be exceeded without a further change in the mechanism. To the mechanism he again recurred, and soon found that it was possible to introduce a change which would cause the action to be completed in three revolutions of the axis. An odd number of revolutions, however, being

attended with certain practical inconveniences, it was considered more advantageous to execute the motions in four turns; and here again the notation put forth its powers, by informing the inventor, *through the eye*, almost independent of his mind, what would be the most elegant, symmetrical, and harmonious disposition of the required motions in four turns. This application of an almost metaphysical system of abstract signs, by which the motion of the hand performs the office of the mind, and of profound practical skill in mechanics alternately, to the construction of a most complicated engine, forcibly reminds us of a parallel in another science, where the chemist with difficulty succeeds in dissolving a refractory mineral, by the alternate action of the most powerful acids, and the most caustic alkalies, repeated in long-continued succession.

This important discovery was explained by Mr Babbage, in a short paper read before the Royal Society, and published in the Philosophical Transactions in 1826.^[15] It is to us more a matter of regret than surprise, that the subject did not receive from scientific men in this country that attention to which its importance in every practical point of view so fully entitled it. To appreciate it would indeed have been scarcely possible, from the very brief memoir which its inventor presented, unaccompanied by any observations or arguments of a nature to force it upon the attention of minds unprepared for it by the nature of their studies or occupations. In this country, science has been generally separated from practical mechanics by a wide chasm. It will be easily admitted, that an assembly of eminent naturalists and physicians, with a sprinkling of astronomers, and one or two abstract mathematicians, were not precisely the persons best qualified to appreciate such an instrument of mechanical investigation as we have here described. We shall not therefore be understood as intending the slightest disrespect for these distinguished persons, when we express our regret, that a discovery of such paramount practical value, in a country preeminently conspicuous for the results of its machinery, should fall still-born and inconsequential through their hands, and be buried unhonoured and undiscriminated in their miscellaneous transactions. We trust that a more auspicious period is at hand; that the chasm which has separated practical from scientific men will speedily close; and that that combination of knowledge will be effected, which can only be obtained when we see the men of science more frequently extending their observant eye over the wonders of our factories, and our great practical manufacturers, with a

reciprocal ambition, presenting themselves as active and useful members of our scientific associations. When this has taken place, an order of scientific men will spring up, which will render impossible an oversight so little creditable to the country as that which has been committed respecting the mechanical notation.^[16] This notation has recently undergone very considerable extension and improvement. An additional section has been introduced into it; designed to express the process of circulation in machines, through which fluids, whether liquid or gaseous, are moved. Mr Babbage, with the assistance of a friend who happened to be conversant with the structure and operation of the steam-engine, has illustrated it with singular felicity and success in its application to that machine. An eminent French surgeon, on seeing the scheme of notation thus applied, immediately suggested the advantages which must attend it as an instrument for expressing the structure, operation, and circulation of the animal system; and we entertain no doubt of its adequacy for that purpose. Not only the mechanical connexion of the solid members of the bodies of men and animals, but likewise the structure and operation of the softer parts, including the muscles, integuments, membranes, &c.; the nature, motion, and circulation of the various fluids, their reciprocal effects, the changes through which they pass, the deposits which they leave in various parts of the system; the functions of respiration, digestion, and assimilation,—all would find appropriate symbols and representatives in the notation, even as it now stands, without those additions of which, however, it is easily susceptible. Indeed, when we reflect for what a very different purpose this scheme of symbols was contrived, we cannot refrain from expressing our wonder that it should seem, in all respects, as if it had been designed expressly for the purposes of anatomy and physiology.

^[15]Phil. Trans. 1820, Part III. p. 250, on a method of expressing by signs the action of machinery.

^[16]This discovery has been more justly appreciated by scientific men abroad. It was, almost immediately after its publication, adopted as the topic of lectures, in an institution on the Continent for the instruction of Civil Engineers.

Another of the uses which the slightest attention to the details of this notation irresistibly forces upon our notice, is to exhibit, in the form of a connected plan or map, the organization of an extensive factory, or any great public institution, in which a vast number of individuals are employed, and their duties regulated (as they generally are or ought to be)

by a consistent and well-digested system. The mechanical notation is admirably adapted, not only to express such an organized connexion of human agents, but even to suggest the improvements of which such organization is susceptible—to betray its weak and defective points, and to disclose, at a glance, the origin of any fault which may, from time to time, be observed in the working of the system. Our limits, however, preclude us from pursuing this interesting topic to the extent which its importance would justify. We shall be satisfied if the hints here thrown out should direct to the subject the attention of those who, being most interested in such an enquiry, are likely to prosecute it with greatest success.

One of the consequences which has arisen in the prosecution of the invention of the calculating machinery, has been the discovery of a multitude of mechanical contrivances, which have been elicited by the exigencies of the undertaking, and which are as novel in their nature as the purposes were novel which they were designed to attain. In some cases several different contrivances were devised for the attainment of the same end; and that among them which was best suited for the purpose was finally selected: the rejected expedients—those overflowings or waste of the invention—were not, however, always found useless. Like the *waste* in various manufactures, they were soon converted to purposes of utility. These rejected contrivances have found their way, in many cases, into the mills of our manufacturers; and we now find them busily effecting purposes, far different from any which the inventor dreamed of, in the spinning-frames of Manchester.^[17]

^[17]An eminent and wealthy retired manufacturer at Manchester assured us, that on the occasion of a visit to London, when he was favoured with a view of the calculating machinery, he found in it mechanical contrivances, which he subsequently introduced with the greatest advantage into his own spinning-machinery.

Another department of mechanical art, which has been enriched by this invention, has been that of *tools*. The great variety of new forms which it was necessary to produce, created the necessity of contriving and constructing a vast number of novel and most valuable tools, by which, with the aid of the lathe, and that alone, the required forms could be given to the different parts of the machinery with all the requisite accuracy.

The idea of calculation by mechanism is not new. Arithmetical instruments, such as the calculating boards of the ancients, on which they made their computations by the aid of counters—the *Abacus*, an instrument for computing by the aid of balls sliding upon parallel rods—the method of calculation invented by Baron Napier, called by him *Rhabdology*, and since called *Napier's bones*—the Swan Pan of the Chinese—and other similar contrivances, among which more particularly may be mentioned the Sliding Rule, of so much use in practical calculations to modern engineers, will occur to every reader: these may more properly be called *arithmetical instruments*, partaking more or less of a mechanical character. But the earliest piece of mechanism to which the name of a 'calculating machine' can fairly be given, appears to have been a machine invented by the celebrated Pascal. This philosopher and mathematician, at a very early age, being engaged with his father, who held an official situation in Upper Normandy, the duties of which required frequent numerical calculations, contrived a piece of mechanism to facilitate the performance of them. This mechanism consisted of a series of wheels, carrying cylindrical barrels, on which were engraved the ten arithmetical characters, in a manner not very dissimilar to that already described. The wheel which expressed each order of units was so connected with the wheel which expressed the superior order, that when the former passed from 9 to 0, the latter was necessarily advanced one figure; and thus the process of carrying was executed by mechanism: when one number was to be added to another by this machine, the addition of each figure to the other was performed by the hand; when it was required to add more than two numbers, the additions were performed in the same manner successively; the second was added to the first, the third to their sum, and so on.

Subtraction was reduced to addition by the method of arithmetical complements; multiplication was performed by a succession of additions; and division by a succession of subtractions. In all cases, however, the operations were executed from wheel to wheel by the hand.^[18]

^[18]See a description of this machine by Diderot, in the *Encyc. Method.*; also in the works of Pascal, tom. IV., p. 7; Paris, 1819.

This mechanism, which was invented about the year 1650, does not appear ever to have been brought into any practical use; and seems to have speedily found its appropriate place in a museum of curiosities. It was

capable of performing only particular arithmetical operations, and these subject to all the chances of error in manipulation; attended also with little more expedition (if so much), as would be attained by the pen of an expert computer.

This attempt of Pascal was followed by various others, with very little improvement, and with no additional success. Polenus, a learned and ingenious Italian, invented a machine by which multiplication was performed, but which does not appear to have afforded any material facilities, nor any more security against error than the common process of the pen. A similar attempt was made by Sir Samuel Moreland, who is described as having transferred to wheel-work the figures of *Napier's bones*, and as having made some additions to the machine of Pascal.^[19]

^[19]Equidem Morelandus in Anglia, tubæ stentoriæ author, Rhabdologiam ex baculis in cylindros transtulit, et additiones auxiliares peragit in adjuncta machina additionum Pascaliana.

Grillet, a French mechanician, made a like attempt with as little success. Another contrivance for mechanical calculation was made by Saunderson. Mechanical contrivances for performing particular arithmetical processes were also made about a century ago by Delepréne and Boitissendeau; but they were merely modifications of Pascal's, without varying or extending its objects. But one of the most remarkable attempts of this kind which has been made since that of Pascal, was a machine invented by Leibnitz, of which we are not aware that any detailed or intelligible description was ever published. Leibnitz described its mode of operation, and its results, in the Berlin Miscellany,^[20] but he appears to have declined any description of its details. In a letter addressed by him to Bernoulli, in answer to a request of the latter that he would afford a description of the machinery, he says, 'Descriptionem ejus dare accuratam res non facilis foret. De effectu ex eo judicaveris quod ad multiplicandum numerum sex figurarum, *e.g.* rotam quamdam tantum sexies gyrari necesse est, nulla alia opera mentis, nullis additionibus intervenientibus; quo facto, integrum absolutumque productum oculis objicietur.'^[21] He goes on to say that the process of division is performed independently of a succession of subtractions, such as that used by Pascal.

^[20]Tom. I., p. 317.

^[21]*Com. Epist.* tom. I., p. 289.

It appears that this machine was one of an extremely complicated nature, which would be attended with considerable expense of construction, and only fit to be used in cases where numerous and expensive calculations were necessary.^[22] Leibnitz observes to his correspondent, who required whether it might not be brought into common use, 'Non est facta pro his qui olera aut pisculos vendunt, sed pro observatoriis aut cameris computorum, aut aliis, qui sumptus facile ferunt et multo calculo egent.' Nevertheless, it does not appear that this contrivance, of which the inventor states that he caused two models to be made, was ever applied to any useful purpose; nor indeed do the mechanical details of the invention appear ever to have been published.

^[22]Sed machinam esse sumptuosam et multarum rotarum instar horologii: Huygenius aliquoties admonuit ut absolvi curarem; quod non sine magno sumptu tædioque factum est, dum varie mihi cum opificibus fuit conflictandum.—*Com. Epist.*

Even had the mechanism of these machines performed all which their inventors expected from them, they would have been still altogether inapplicable for the purposes to which it is proposed that the calculating machinery of Mr Babbage shall be applied. They were all constructed with a view to perform particular arithmetical operations, and in all of them the accuracy of the result depended more or less upon manipulation. The principle of the calculating machinery of Mr Babbage is perfectly general in its nature, not depending on any *particular arithmetical operation*, and is equally applicable to numerical tables of every kind. This distinguishing characteristic was well expressed by Mr Colebrooke in his address to the Astronomical Society on this invention. 'The principle which essentially distinguishes Mr Babbage's invention from all these is, that it proposes to calculate a series of numbers following any law, by the aid of differences, and that by setting a few figures at the outset; a long series of numbers is readily produced by a mechanical operation. The method of differences in a very wide sense is the mathematical principle of the contrivance. A machine to add a number of arbitrary figures together is no economy of time or trouble, since each individual figure must be placed in the machine; but it is otherwise when those figures follow some law. The insertion of a few at first determines the magnitude of the next, and those of the succeeding. It is this constant repetition of similar operations which renders the computation of tables a fit subject for the application of machinery. Mr Babbage's

invention puts an engine in the place of the computer; the question is set to the instrument, or the instrument is set to the question, and by simply giving it motion the solution is wrought, and a string of answers is exhibited.' But perhaps the greatest of its advantages is, that it prints what it calculates; and this completely precludes the possibility of error in those numerical results which pass into the hands of the public. 'The usefulness of the instrument,' says Mr Colebrooke, 'is thus more than doubled; for it not only saves time and trouble in transcribing results into a tabular form, and setting types for the printing of the table, but it likewise accomplishes the yet more important object of ensuring accuracy, obviating numerous sources of error through the careless hands of transcribers and compositors.'

Some solicitude will doubtless be felt respecting the present state of the calculating machinery, and the probable period of its completion. In the beginning of the year 1829, Government directed the Royal Society to institute such enquiries as would enable them to report upon the state to which it had then arrived; and also whether the progress made in its construction confirmed them in the opinion which they had formerly expressed,—that it would ultimately prove adequate to the important object which it was intended to attain. The Royal Society, in accordance with these directions, appointed a Committee to make the necessary enquiry, and report. This Committee consisted of Mr Davies Gilbert, then President, the Secretaries, Sir John Herschel, Mr Francis Baily, Mr Brunel, engineer, Mr Donkin, engineer, Mr G. Rennie, engineer, Mr Barton, comptroller of the Mint, and Mr Warburton, M.P. The voluminous drawings, the various tools, and the portion of the machinery then executed, underwent a close and elaborate examination by this Committee, who reported upon it to the Society.

They stated in their report, that they declined the consideration of the principle on which the practicability of the machinery depends, and of the public utility of the object which it proposes to attain; because they considered the former fully admitted, and the latter obvious to all who consider the immense advantage of accurate numerical tables in all matters of calculation, especially in those which relate to astronomy and navigation, and the great variety and extent of those which it is professedly the object

of the machinery to calculate and print with perfect accuracy;—that absolute accuracy being one of the prominent pretensions of the undertaking, they had directed their attention especially to this point, by careful examination of the drawings and of the work already executed, and by repeated conferences with Mr Babbage on the subject;—that the result of their enquiry was, that such precautions appeared to have been taken in every part of the contrivance, and so fully aware was the inventor of every circumstance which might by possibility produce error, that they had no hesitation in stating their belief that these precautions were effectual, and that whatever the machine would do, it would do truly.

They further stated, that the progress which Mr Babbage had then made, considering the very great difficulties to be overcome in an undertaking of so novel a kind, fully equalled any expectations that could reasonably have been formed; and that although several years had elapsed since the commencement of the undertaking, yet when the necessity of constructing plans, sections, elevations, and working drawings of every part; of constructing, and in many cases inventing, tools and machinery of great expense and complexity, necessary to form with the requisite precision parts of the apparatus differing from any which had previously been introduced in ordinary mechanical works; of making many trials to ascertain the value of each proposed contrivance; of altering, improving, and simplifying the drawings;—that, considering all these matters, the Committee, instead of feeling surprise at the time which the work has occupied, felt more disposed to wonder at the possibility of accomplishing so much.

The Committee expressed their confident opinion of the adequacy of the machinery to work under all the friction and strain to which it can be exposed; of its durability, strength, solidity, and equilibrium; of the prevention of, or compensation for, wear by friction; of the accuracy of the various adjustments; and of the judgment and discretion displayed by the inventor, in his determination to admit into the mechanism nothing but the very best and most finished workmanship; as a contrary course would have been false economy, and might have led to the loss of the whole capital expended on it.

Finally, considering all that had come before them, and relying on the talent and skill displayed by Mr Babbage as a mechanist in the progress of this arduous undertaking, not less for what remained, than on the matured

and digested plan and admirable execution of what is completed, the Committee did not hesitate to express their opinion, that in the then state of the engine, they regarded it as likely to fulfil the expectations entertained of it by its inventor.

This report was printed in the commencement of the year 1829. From that time until the beginning of the year 1833, the progress of the work has been slow and interrupted. Meanwhile many unfounded rumours have obtained circulation as to the course adopted by Government in this undertaking; and as to the position in which Mr Babbage stands with respect to it. We shall here state, upon authority on which the most perfect reliance may be placed, what have been the actual circumstances of the arrangement which has been made, and of the steps which have been already taken.

Being advised that the objects of the projected machinery were of paramount national importance to a maritime country, and that, from its nature, it could never be undertaken with advantage by any individual as a pecuniary speculation, Government determined to engage Mr Babbage to construct the calculating engine for the nation. It was then thought that the work could be completed in two or three years; and it was accordingly undertaken on this understanding about the year 1821, and since then has been in progress. The execution of the workmanship was confided to an engineer by whom all the subordinate workmen were employed, and who supplied for the work the requisite tools and other machinery; the latter being his own property, and not that of Government. This engineer furnished, at intervals, his accounts, which were duly audited by proper persons appointed for that purpose. It was thought advisable—with a view, perhaps, to invest Mr Babbage with a more strict authority over the subordinate agents—that the payments of these accounts of the engineer should pass through his hands. The amount was accordingly from time to time issued to him by the Treasury, and paid over to the engineer. This circumstance has given rise to reports, that he has received considerable sums of money as a remuneration for his skill and labour in inventing and constructing this machinery. Such reports are altogether destitute of truth. He has received, neither directly nor indirectly, any remuneration whatever;—on the contrary, owing to various official delays in the issues of money from the Treasury for the payment of the engineer, he has frequently been

obliged to advance these payments himself, that the work might proceed without interruption. Had he not been enabled to do this from his private resources, it would have been impossible that the machinery could have arrived at its present advanced state.

It will be a matter of regret to every friend of science to learn, that, notwithstanding such assistance, the progress of the work has been suspended, and the workmen dismissed for more than a year and a half; nor does there at the present moment appear to be any immediate prospect of its being resumed. What the causes may be of a suspension so extraordinary, of a project of such great national and universal interest,—in which the country has already invested a sum of such serious amount as L.15,000,—is a question which will at once suggest itself to every mind; and is one to which, notwithstanding frequent enquiries, in quarters from which correct information might be expected, we have not been able to obtain any satisfactory answer. It is not true, we are assured, that the Government object to make the necessary payments, or even advances, to carry on the work. It is not true, we also are assured, that any practical difficulty has arisen in the construction of the mechanism;—on the contrary, the drawings of all the parts of it are completed, and may be inspected by any person appointed on the part of Government to examine them.^[23] Mr Babbage is known as a man of unwearied activity, and aspiring ambition. Why, then, it may be asked, is it that he, seeing his present reputation and future fame depending in so great a degree upon the successful issue of this undertaking, has nevertheless allowed it to stand still for so long a period, without distinctly pointing out to Government the course which they should adopt to remove the causes of delay? Had he done this (which we consider to be equally due to the nation and to himself), he would have thrown upon Government and its agents the whole responsibility for the delay and consequent loss; but we believe he has not done so. On the contrary, it is said that he has of late almost withdrawn from all interference on the subject, either with the Government or the engineer. Does not Mr Babbage perceive the inference which the world will draw from this course of conduct? Does he not see that they will impute it to a distrust of his own power, or even to a consciousness of his own inability to complete what he has begun? We feel assured that such is not the case; and we are anxious, equally for the sake of science, and for Mr Babbage's own reputation, that the mystery—for such it must be regarded—should be cleared up; and that

all obstructions to the progress of the undertaking should immediately be removed. Does this supineness and apparent indifference, so incompatible with the known character of Mr Babbage, arise from any feeling of dissatisfaction at the existing arrangements between himself and the Government? If such be the actual cause of the delay, (and we believe that, in some degree, it is so,) we cannot refrain from expressing our surprise that he does not adopt the candid and straightforward course of declaring the grounds of his discontent, and explaining the arrangement which he desires to be adopted. We do not hesitate to say, that every reasonable accommodation and assistance ought to be afforded him. But if he will pertinaciously abstain from this, to our minds, obvious and proper course, then it is surely the duty of Government to appoint proper persons to enquire into and report on the present state of the machinery; to ascertain the causes of its suspension; and to recommend such measures as may appear to be most effectual to ensure its speedy completion. If they do not by such means succeed in putting the project in a state of advancement, they will at least shift from themselves all responsibility for its suspension.

[23] Government has erected a fire-proof building, in which it is intended that the calculating machinery shall be placed when completed. In this building are now deposited the large collection of drawings, containing the designs, not only of the part of the machinery which has been already constructed, but what is of much greater importance, of those parts which have not yet been even modelled. It is gratifying to know that Government has shown a proper solicitude for the preservation of those precious but perishable documents, the loss or destruction of which would, in the event of the death of the inventor, render the completion of the machinery impracticable.

*** END OF THE PROJECT GUTENBERG EBOOK THE
CALCULATING ENGINE ***

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